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Numerical Analysis of thin-walled Prismatic Columns Subjected to Low Velocity Axial Impact Load

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Abstract

Thin-wall prismatic columns are used to absorb impact energy in various engineering systems, e.g. the frontal chassis structure of an automobile. This paper presents numerical analyses of thin-walled prismatic columns having various cross-sections (square, rectangular, hexagon, octagon and circular) subjected to low velocity axial impact loads using a non-linear finite element method. In this work, the behavior of these column is absorbing energy is compared. It is shown that octagonal and circular column have the highest mean crushing force whereas square column has lowest mean crushing force.

Keywords: Crashworthiness; Thin-walled column; Axial crushing

1. Introduction

In automotive industry, impact safety and weight-saving are two important design goals for energy absorbing structure. Box structure has an important role in absorbing energy of automobiles, e.g. the frontal chassis structure of automobile, bumpers. Due to the importance of this, a number of researchers have conducted experimental and numerical studies investigating the crashworthiness of various types of structures. Many researchers have focused their attention on the performance of axial crushing of square tube [1-2] due to their manufacturability, common application in space frames and energy absorption capability.

A review of literature showed that little information was available on the behavior of thin-walled sections under crushing load in general and, especially dynamic crushing load. Furthermore, very little numerical work has been carried out on thin-walled sections. So that, this research focused on the effect of cross-section geometry on the energy absorbing properties of thin-walled prismatic column subjected to low velocity axial impact load using finite element analysis.

In this research, the deformation behavior, load-displacement curves and energy absorption were studied. Numerical simulations which based on the finite element method using the explicit non-linear code LS-DYNA are efficient and reliable for investigation of material and component behavior. In order to be able to initiate buckling phenomenon, geometric imperfection should be introduced in the model. In this work, the imperfection (trigger) was modeled as a circumferential slot located at the theoretical full folding length measured from the top of the tube. And the final point is finding out the cross-section with the best capability in absorbing impact energy.

2. Crushing behavior of prismatic column

2.1 Crushing of a circular tube

The crushing theory of a circular tube, firstly developed by Alexander [3], and then improved by Abramowicz and Norman Jones [4]. Consider a circular tube with radius *R* and thickness *t* undergoing axial crushing as shown in Figure 1. The plastic energy is dissipated through the formation of one completed wrinkle. It concludes total plastic energy absorbed by hinge a & c, total energy dissipated by hinge b and Energy absorbed in circumferential stretching during $\phi \rightarrow \phi + d\phi$.

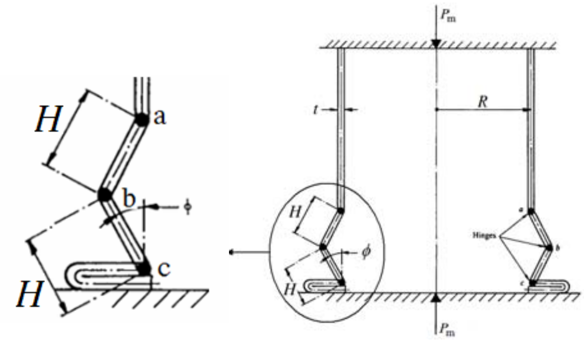


Figure 1. Idealized axisymmetric, or concentric, crushing mode for an axially compressed cylindrical shell.

<u>In case of without considering effective crushing distance</u>

The half folding element H

$$\frac{H}{R}=1.76\left(\frac{t}{2R}\right)^{\frac{1}{2}}$$ for axisymmetric crushing mode (1)

$$\frac{H}{R}=0.816\left(\frac{t}{2R}\right)^{\frac{1}{3}}$$ for non-axisymmetric (dimond) crushing mode (2)

The mean crushing force is given by

$$\frac{P_m}{M_0}=20.79\left(\frac{2R}{t}\right)^{\frac{1}{2}}+11.90$$ for axisymmetric crushing mode (3)

$$\frac{P_m}{M_0}=62.88\left(\frac{2R}{t}\right)^{\frac{1}{3}}$$ for non-axisymmetric (dimond) crushing mode (4)

<u>In case of considering effective crushing distance</u>

The mean crushing force is given by

$$\frac{\overline{P}_m}{M_0}=\frac{20.79\left(\frac{2\mathrm{R}}{\mathrm{t}}\right)^{\frac{1}{2}}+11.90}{0.86-0.568\left(\frac{t}{2R}\right)^{\frac{1}{2}}}$$ for axisymmetric crushing mode (5)

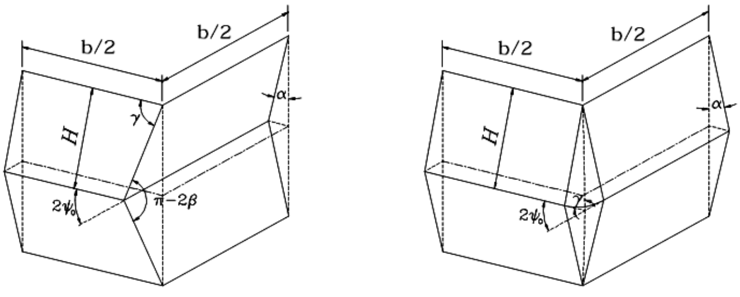
$$\frac{\overline{P}_m}{M_0}=86.14\left(\frac{2R}{t}\right)^{\frac{1}{3}}$$ for non-axisymmetric (dimond) crushing mode (6)

Where M_0 is the plastic collapse moment for the cross-section (per unit circumferential length)

$$M_0=\left(\frac{2\sigma_0}{\sqrt{3}}\right)\frac{t^2}{4} \tag{7}$$

2.2 Thin-walled column crushing resistance

Solution for the crushing resistance of multi-corner sheet metal columns is based on a concept of superfolding (Figure 2) develop by Weirzbicky and Abramowicz (1983) [5]. The corresponding theory for a right angle element was later extended to multi-corner, arbitrarily shaped column by Abramowicz and Weirzbicky (1989) [6] (Figure 2).



(a) (b)

Figure 2. Quasi-inextensional (a) and extensional (b) folding mechanisms

The present method can be easily used to predict the crushing resistance of prismatic columns (square, hexagonal, octagonal, etc.)

Square column ($\psi_0 = \frac{\pi}{4}$)

Consider a square tube with cross-section of *b* x *b* and thickness *t* undergoing axial crushing as show in Figure 3. The plastic deformation is localized at a portion of the column and the plastic energy is dissipated through the deformation of hinge lines and membrane action zones. The localized plastic deformation zone is defined as the superfolding element. The folding element is characterized by the half folding element *H*, which is obtained from the condition of minimum mean crushing force (Weirzbicky and Abramowicz 1983).

$$H = 0.93t^{1/3}b^{2/3} \tag{8}$$

The mean crushing force is given by

$$P_m = 13.05\sigma_0 t^{5/3} b^{1/3} \tag{9}$$

where σ_0 is the flow stress of the column material

$$\sigma_0 = \sqrt[3]{\frac{2\sigma_y \sigma_u^2}{(n+1)^2 (n+2)}} \tag{10}$$

where σ_y and σ_u are the yield and ultimate strength, and n is the train hardening exponent of the thin-walled material. σ_0 for progressively collapsing prismatic columns made from mild steel equals approximately to: $\sigma_0 = 0.92\sigma_u$

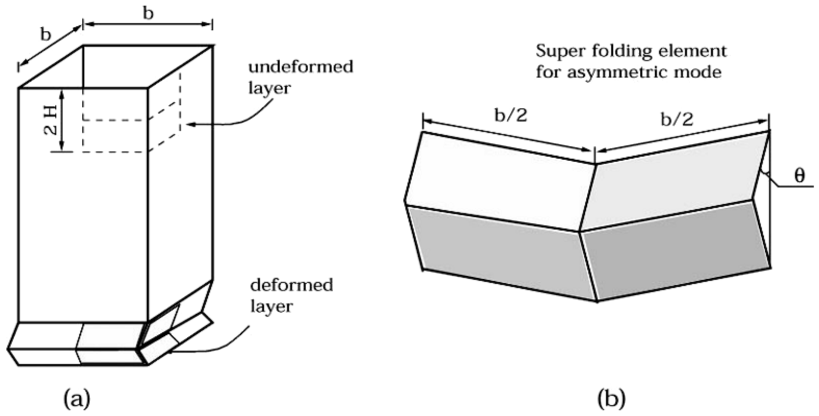


Figure 3. (a) Deformation pattern of box column, (b) supperfolding element

Hexagonal column ($\psi_0 = \frac{\pi}{6}$)

The half folding element H

$$H = 0.63t^{1/3}b^{2/3} \tag{11}$$

The mean crushing force is given by

$$P_m = 20.23\sigma_0 t^{5/3}b^{1/3} \tag{12}$$

Octagonal column ($\psi_0 = \frac{\pi}{8}$)

The half folding element H

$$H = 0.54t^{1/3}b^{2/3} \tag{13}$$

The mean crushing force is given by

$$P_m = 34.82\sigma_0 t^{5/3}b^{1/3} \tag{14}$$

3. Finite element modelling

In this study, an explicit non-linear commercial finite element code LS-DYNA 970 was used to predict the response of the thin-walled structures subjected to axial crushing, dynamic loading conditions with large strain analysis. Full 3-dimensional model is shown in Figure 4.

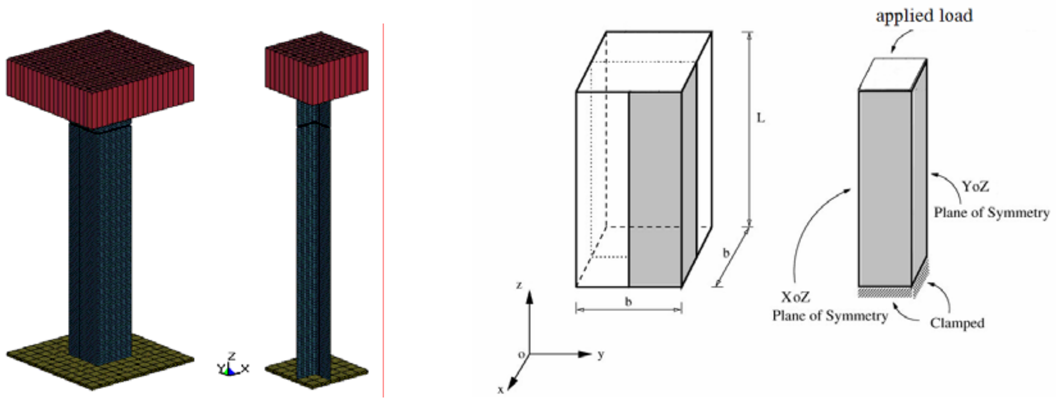


Figure 4. Geometric modeling of square tube

3.1 Discretisation of model

The specimens were modeled with a Belytschoko-Tsay shell element with four nodes. Since the aim of this research is to find out the tube having with the best capability in absorbing impact energy with same weight criteria. The geometry of the square tube (L = 250 mm; b = 40 mm and t = 1.50 mm, t = 1.88 mm) is used as a reference weight to calculate geometry of the rest (Table 1)

Table 1. Geometry of the tube

	Square tube	**Hexagonal tube**	**Octagonal tube**	**Circular tube**
***b* (mm)**	40.00	26.67	20	*R* (mm) = 25.48

From experience when working with impact prismatic column, it is found that an element size of approximately 2 mm is appropriate for modeling the axial crushing of box columns. Therefore, this size was used in this research given a total of 2880 shell elements for a 250 mm length, 40 mm wide columns. Hourglass control #5 is used to eliminate spurious zero-energy modes in the solution.

A default eight-node solid element was used to model the impactor and supporter.

3.2 *Boundary and interface contact condition*

Deformation of the tubes has two symmetry planes with respect to their cross section, i.e. YOZ and XOZ. Due to the expected symmetry of the deformation, only one quarter of the column was modeled to represent the axial crushing problem. The lower end of the specimens was clamped; the upper end was stroked by impactor (M = 11 kg) with velocity 9.5 m/s for dynamic crushing.

Several different contact algorithms available in LS-DYNA were used. The contact between rigid striker and tube, tube and supporter were modeled with an automatic node to surface. Automatic single surface contact was applied to the column walls to avoid interpenetration of folds generated during axial collapse. Values for the coefficients of static and dynamic friction were specified as 0.4 and 0.3 respectively.

3.3 *Triggering mechanism*

In order to be able to initiate buckling phenomenon, geometric imperfection should be introduced in the model. In this work, the imperfection (trigger) was modeled as a circumferential slot located at the theoretical full folding length 2H measured from the top of the tube, with H is different from kind of tubes.

3.4 *Constitutive modeling of materials*

The constitutive behavior of the thin shell element for the column material was based on the elastic-plastic material model (Mat_Piecewise_Linear_Plasicity) with Von Mises's isotropic plasticity algorithm. The transverse shear stress was considered by this material model. Plastic hardening was based on the polygonal curve definition, in which pairs of the plastic tangent modulus and the plastic stress were specified.

The steel column profile was made of mild steel RSt37 with mechanical properties: density $\rho = 7830$ kg/m^3, Young's modulus E = 2.10^5 N/mm^2, initial yield stress σ_y = 251 N/mm^2, Poisson's ratio ν = 0.3, and the power law exponent n = 0.12. The engineering stress-strain for material was given in Table 2.

Table 2. Strain hardening data for Mild Steel RSt37

Plastic strain (%)	0.0	2.4	4.9	7.4	9.9	12.4	14.9	17.4
RSt37 Plastic stress (MPa)	251	264	295	316	326	334	336	339

4. Nummerical analysis

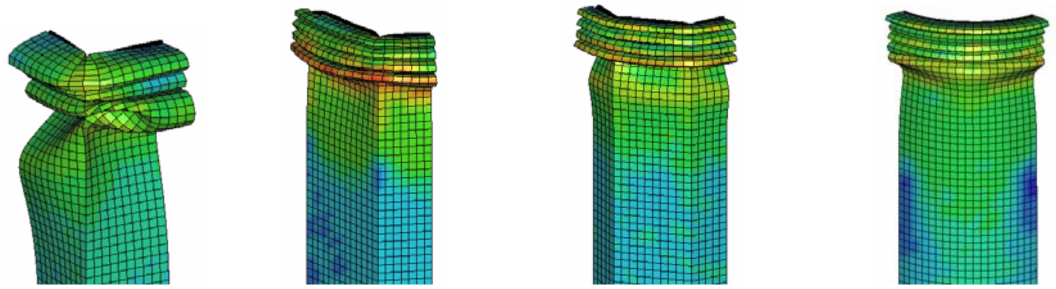
This research focused to simulate the prismatic column made of mild steel RSt37 with the cross section width based on the one of square column b = 40 mm and thickness (t = 1.5mm & t = 1.88 mm). The numerical analyzed with respect to the plastic folding mode, instantaneous and mean crushing force.

4.1 *Plastic folding mode*

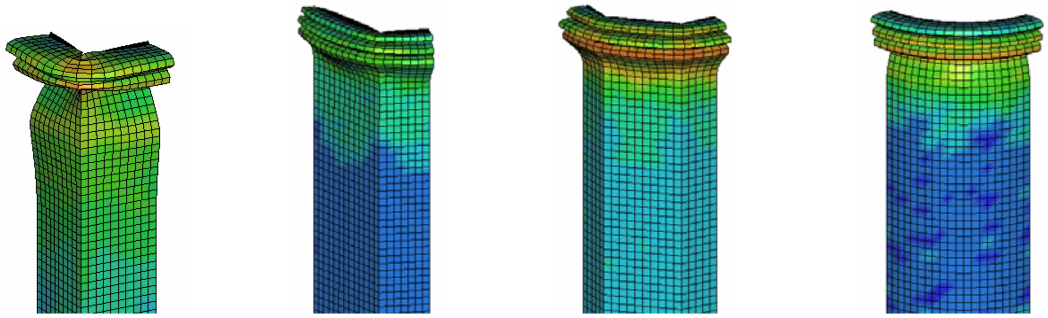
The deformation patterns of the prismatic columns (square tube, hexagonal tube, octagonal tube and circular tube) were shown in Figure 5. Almost the column walls deform progressive by forming outward folds which is referred to extensional mode. Especially, the last folding of square tube *t* = 1.50 mm is in-extensional mode, there is the transition from extensional mode to in-extensional mode. The number of lobes which were formed due to plastic folding was different from type of tube and the crushing distance is not the same also. These are main reasons lead to the difference of tubes energy absorbing. Half-wave plastic folding of four kinds of tube can be calculated by using Eq. (1), (8), (11) and (13), were given in Table 3. Furthermore, from numerical simulation, Figure 6, it is easy to see that the theoretical prediction to calculate the plastic folding length is in good agreement with the numerical simulation.

Table 3. Full plastic folding length of four kinds of tube

	Square tube	**Hexagonal tube**	**Octagonal tube**	**Circular tube**
***t* = 1.50 mm**	25.0 mm	13.0 mm	9.1 mm	15.4 mm
***t* = 1.88 mm**	26.8 mm	14.0 mm	10.0 mm	17.2 mm



(a) Square tube Hexagonal tube Octagonal tube Circular tube



(b) Square tube Hexagonal tube Octagonal tube Circular tube

Figure 5. Deformation pattern of the prismatic columns (RSt37), (a) *t* = 1.50 mm; (b) *t* = 1.88 mm

4.2 Instantaneous and mean crushing force

The instantaneous and mean crushing force of prismatic columns, *t* = 1.50 mm and *t* = 1.88 mm, were showed in Figure 6 and Figure 7. The peak force in the instantaneous crushing force curve corresponds to the progressive fold formation, while the distance between two consecutive peaks corresponds to the plastic folding length.

The mean crushing force is defined

$$P_m = \frac{1}{\delta}\int_0^{\delta} P(\delta)\,d\delta \tag{15}$$

Where $P(\delta)$ is the instantaneous crushing load corresponding to the instantaneous shortening δ. The instantaneous crushing load can be obtained from the numerical simulation.

In order to predict the mean crushing force, using theory analytical equations (5), (9), (12) and (14) give result in Table 4. After that, make a comparison between theory analysis and numerical analysis which are shown in Figure 6 and Figure 7. This findings show that, in case of square tube and circular tube, theoretical prediction to calculate the mean crushing force is in a very good agreement with the numerical simulation. There is approximated 7% error theory analysis larger than numerical simulation in case of hexagonal tube. However, the error of octagonal tube behavior cannot be acceptable if using theory based on a concept of superfolding. An interesting result of octagonal tube crushing behavior is so close to behavior of circular tube. With this reason, it is considered that the crushing theory of a circular tube can be used to predict the behavior of octagonal tube crushing.

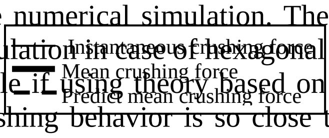
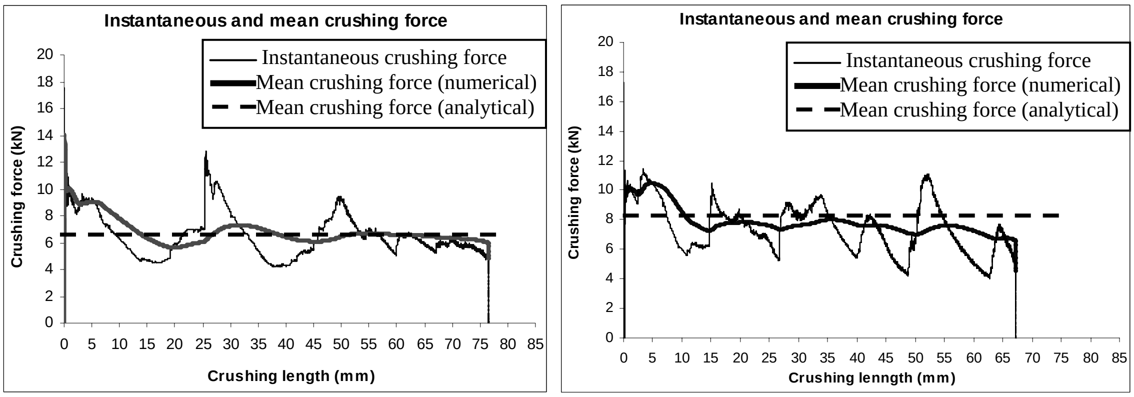


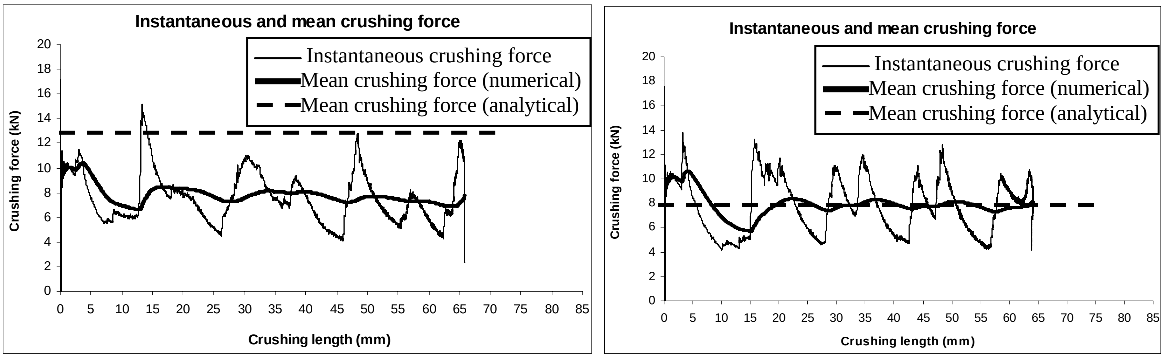
Table 4. Mean crushing force using theory analytical equation for the prismatic column (t = 1.50 mm)

	Square tube	**Hexagonal tube**	**Octagonal tube**	**Circular tube**
P_m (kN) (t = 1.50 mm)	6.11	8.28	12.95	7.97
P_m (kN) (t = 1.88 mm)	8.90	12.03	18.87	11.38



Square tube

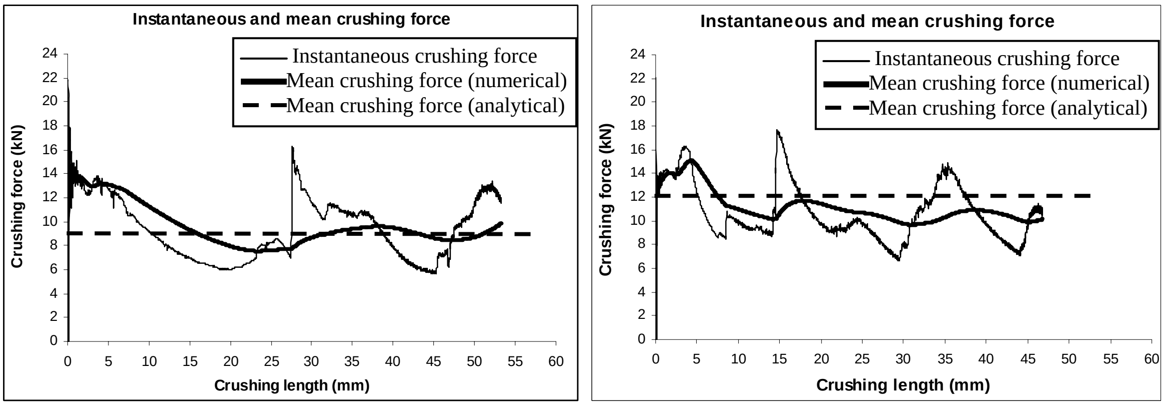
Hexagonal tube



Octagonal tube

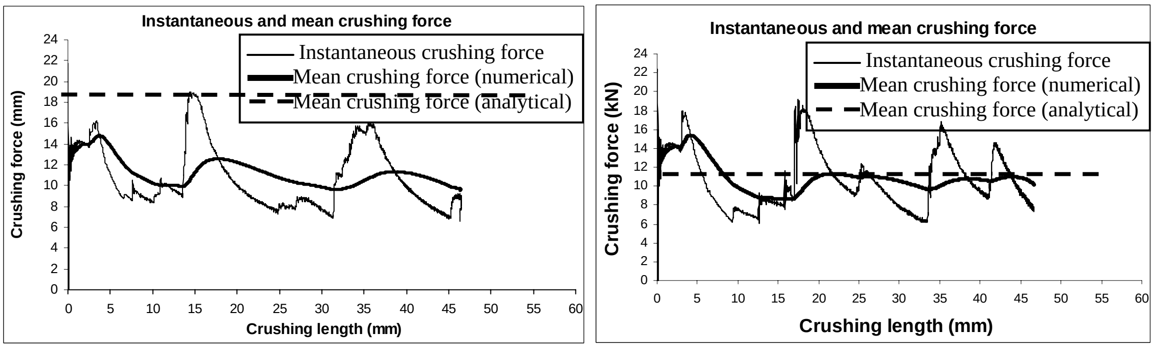
Circular tube

Figure 6. Instantaneous and mean crushing force of prismatic column with t = 1.50 mm



Square tube

Hexagonal tube

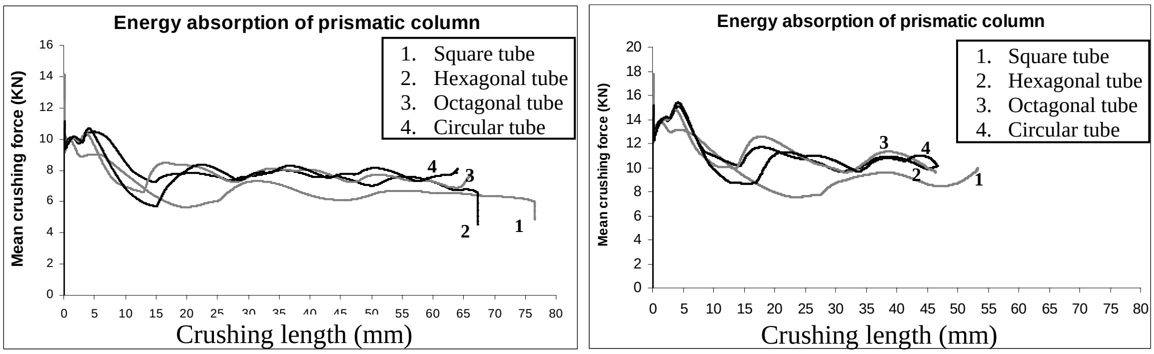


Octagonal tube Circular tube

Figure 7. Instantaneous and mean crushing force of prismatic column with $t = 1.88$ mm

4.3 *Comparison the energy absorption capability*

The aim of this research is to compare the behavior of prismatic column is absorbing energy. The results in Figure 8 are shown that octagonal and circular column have the highest mean crushing force, lowest crushing length while square column has lowest mean crushing force but highest crushing length. So, depend upon purpose of using prismatic column to absorb energy, in which high energy absorption or low mean crushing force is priority, we can give a wise decision.



(a) (b)

Figure 8. Energy absorption of prismatic column; (a): tube with $t = 1.50$ mm; (b): tube with $t = 1.88$ mm

4.4 *Effect of column geometry*

Table 4 shows that when thickness of the prismatic column is increased 0.38 mm, 25.3% compare to old thickness, mean crushing force is increased 45% and crushing length is reduced also. It leads to energy absorption capability increases significantly. However, it is not wise if increasing too much thickness of tube because it causes structures much heavier, in this case people tend to use foam-filled prismatic column.

5. Conclusion

The numerical simulation conducted in this research has provided a significant amount of information regarding the energy absorption capability. The collapse profiles and mean crushing force of prismatic column subjected to low velocity impact load have been able to be predicted both analytically and numerically. Simulation result shows that by increasing the number of corner, the crushing behavior of prismatic column will approach the behavior of circular tube. Octagonal and circular column have the highest mean crushing force whereas square column has lowest mean crushing force.

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