Stress Concentration Factors of Stepped-Shafts of Circular-to-Square Cross Section under Pure Bending

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Abstract

Shaft is a machine element whose main function for power transmission. Shaft is often needed in stepped form (change in cross-section). For a non-smooth shaft (stepped shaft), the elementary stress equations are no longer hold for calculating the actual stress as the stress would be higher than the nominal stresses. To relate the maximum stress at the discontinuity with the nominal stress calculated using the elementary stress equation, a factor is needed which is called stress concentration factor (SCF). This paper presents a graph for design purpose of stress concentration factor of stepped shaft of circular-to-square cross-section under pure bending. The graph was derived from an extensive linier finite element parametric study and presented in line of the well-known Peterson's graph for stepped shaft of uniform circular cross-section. Finite element parametric study was carried out for every combination of diameter to diagonal ratio, D/d and radius of fillet to diagonal ratio, r/d. For geometry with particular D/d value, six analyses were done by varying the r/d value. Stress distribution shows that maximum axial (bending) stress located at the root of square, exactly at the corner. In general graphical design similar to the well known Peterson graph was produced, but with small range of parameter because of geometry restriction.

Keywords: stepped-shafts, stress-concentration factors, finite element analysis

Introduction

Shaft is a machine element whose function for power transmission. Shaft is often needed in stepped form (change in cross-section). Example is a rotated shaft with square shoulder for the seat of bearing. In such, the shaft could be subjected to combined loading of axial, bending, and torsion.

If a shaft is smooth, the resulting stress due to loading can be computed using the elementary stress equations. This stress is called nominal stress. For a non-smooth shaft (stepped shaft), the elementary stress equations are no longer hold for calculating the actual stress as the stress would be higher than the nominal stresses. To relate the maximum stress at the discontinuity with the nominal stress calculated using the elementary stress equation, a factor is needed which is called stress concentration factor (SCF). The magnitude of this factor depends on diameter ratio, shape and area of cross-section, radius of fillet and types of loading (axial, bending, torsion, or their combination).

Stress concentration factor for a stepped shaft of uniform cross-section can be found in many standard textbooks of mechanics of materials. Until now, the most complete source of graph of stress concentration factors for various geometry of shaft, bar, and plat is a compendia by Peterson [1974]. In this compendia, stress concentration factor (K_t) for stepped shaft of uniform cross-section are plotted as function of non-dimensional geometry. Today state of the art modern literature are also still using the Peterson's graph for analyzing the stepped shaft of uniform cross-section, see for example, Kang, et al [2005]. Pan, et al [2000] used the Peterson's graph to analyze the torsional vibration of stepped shaft. Kim and Kim [1995] used the same graph to analyze the stresses at the stepped shaft subjected to axial tension.

In the compendia of stress concentration factors by Peterson, it cannot be found a graph for stepped shaft of circular-to-square cross-section. Baker [1999] carried out a finite element study using ALGOR version 3.18 to obtain the characteristic of stepped shaft of circular to square cross-section subjected to combined loading of static bending and torsion. In his MSc thesis, Baker investigated three values of diameter ratio, but did not make the radius of filled varied so that graphs that can be used for design purpose were not produced. To analyze a stepped shaft of circular-to-square cross-section, it was common to apply the graph for stepped shaft of uniform circular cross-section of equivalent cross-sectional area to square [Yusron, 2004]. This is usually done because similar graph for stepped shaft of circular-to-square cross-section was not available either in textbooks or in today state of the art modern literature. This paper presents a graph for design purpose of stress concentration factor of stepped shaft of circular-to-square cross-section under static pure bending. The graph was derived from an extensive linier finite element parametric study and presented in line of the well-known Peterson's graph for stepped shaft of uniform circular cross-section.

Stress-Concentration Factors

Shaft is an important element in mechanical construction. Shaft is usually has function for power transmission. Shaft can be found as square cross-section as well as circular cross section, however, majority of shaft are circular cross-section as its main function is for power transmission. Whereas, shaft of square cross-section is usually only used for supporting load.

Circular shaft is a rotating element where other elements such as gears and pulley are mounted. Such a shaft could be subjected to axial (tension or compression), bending, torsion, or their combination as shown in Figure 1.

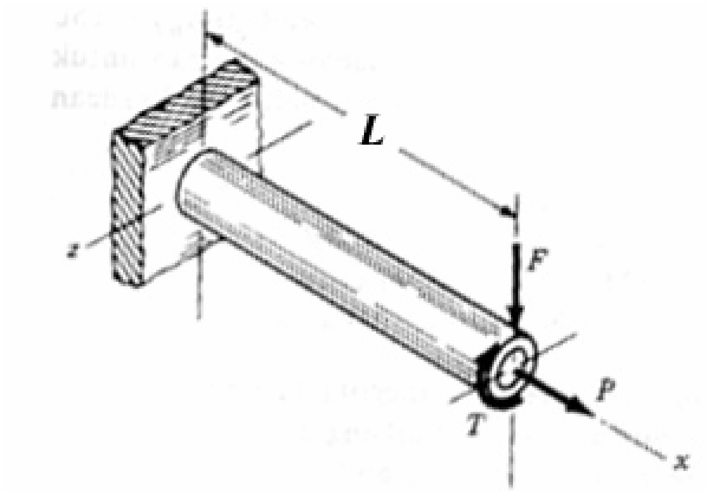

Figure 1 A circular shaft subjected to combined loading of axial, bending and torsion

In particular application, a shaft of non-uniform area of cross section (stepped shaft) might be used. Change in area of cross-section of stepped shaft results in stress concentration at the discontinuity. In order to reduce the effect of stress concentration at the region of change in cross-section, fillet of radius *r* is usually applied as shown in Figure 2.

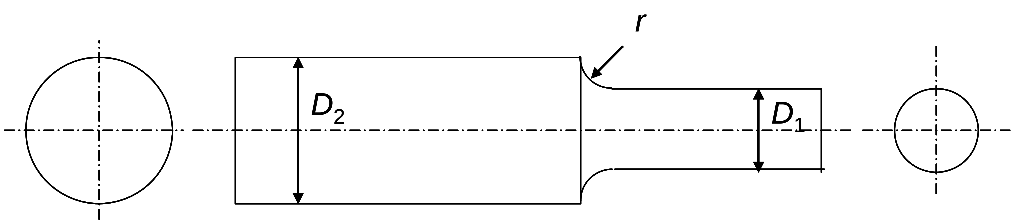

Figure 2 Circular stepped shaft with shoulder or radius *r*

The ratio of maximum stress at a critical point to the nominal stress is called stress concentration factor (*SCF*). Stress concentration factor of circular stepped shaft has been given by Peterson [1974]. The magnitude of this factor is a function of non-dimensional parameter D_2/D_1 and r/D_1. Stress concentration factor, K_t, can be written as:

$$K_t = \frac{\sigma_{\max}}{\sigma_{nom}} \text{; for normal stress} \quad (1a)$$

$$K_{ts} = \frac{\tau_{\max}}{\tau_{nom}} \text{; for shear stress} \quad (1b)$$

Where, for circular cross section the nominal stress is given by:

$$\sigma_{nom} = \frac{4P}{\pi D_1^2} \text{; for axial load} \quad (2)$$

$$\sigma_{nom} = \frac{32M}{\pi D_1^3} \text{; for bending load} \quad (3)$$

$$\tau_{nom} = \frac{16T}{\pi D_1^3} \text{; for torsion load} \quad (4)$$

Figure 3 and 4 are typical graph of stress concentration factor for square and circular stepped-shaft respectively under pure bending load. These graphs were taken from a book of "Stress Concentration Factor" by Peterson [1974]. Similar graph for axial and torsion load can be obtained from the same book.

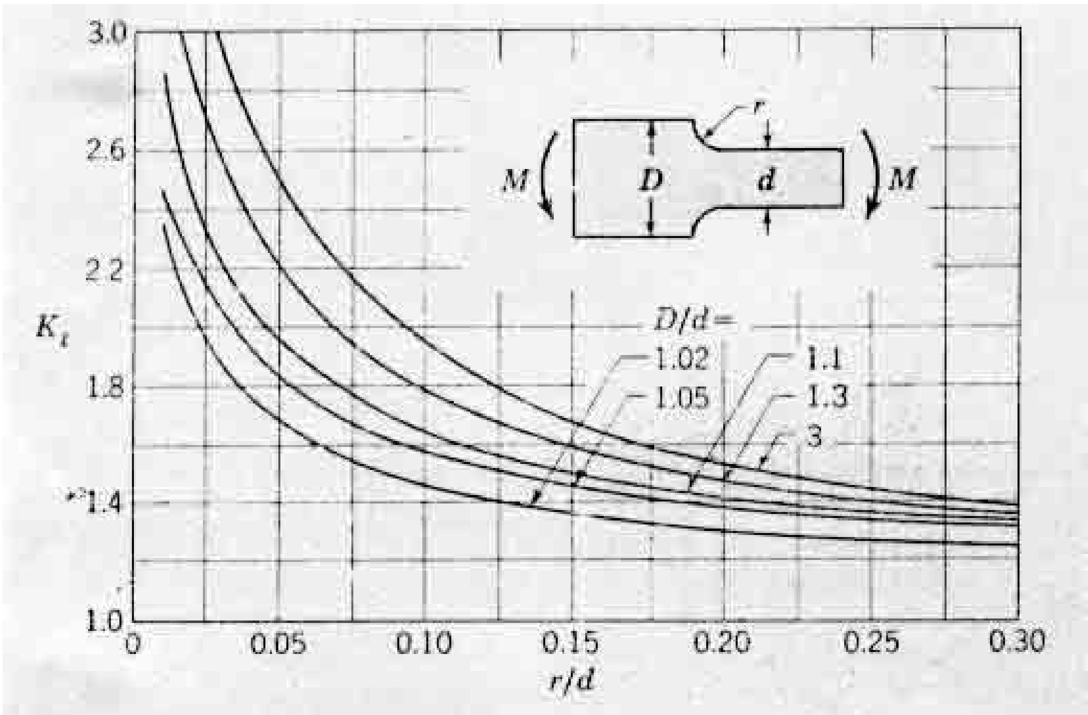

Figure 3 Stress-concentration factors for square stepped-shaft under pure bending [Peterson, 1974]

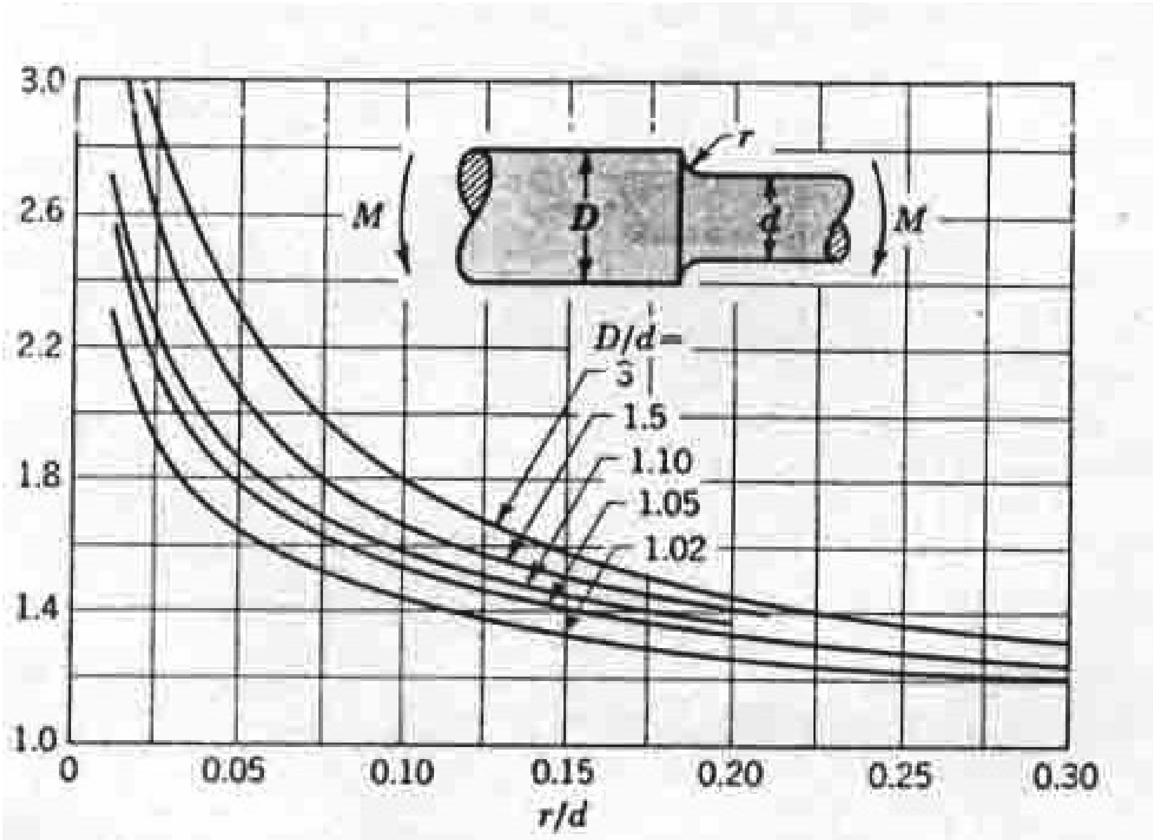

Figure 4 Stress-concentration factors for circular stepped-shaft under pure bending [Peterson, 1974]

In the book of Peterson, it cannot be found a graph of stress-concentration factor for a stepped shaft of circular-to-square cross-section (Figure 5). In fact, stepped shaft of this form can be found in practice, such as, the output low speed reducer at mill and diffuser station in sugar cane industry [Yusron, 2004]. This paper aims to provide a graph of stress concentration factors of stepped shafts of circular to square cross-section under pure bending that can be used directly for design purpose.

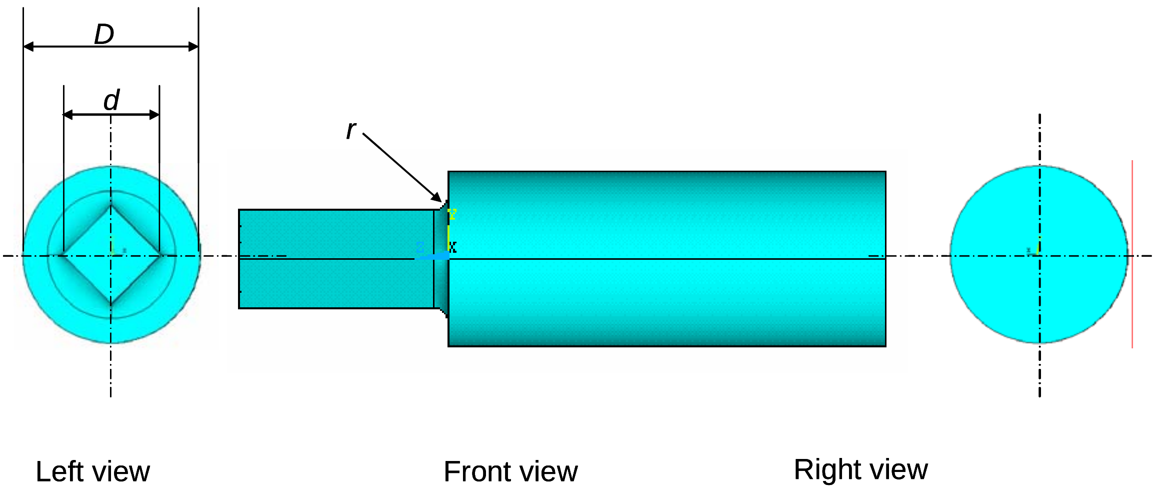

Figure 5 Stepped shaft of circular to square cross-section

Finite Element Analysis

Figure 5 shows a stepped shaft of circular cross section of diameter *D* to square cross-section of diagonal *d*. Finite element parametric study was carried out for every combination of parameter *D/d* and *r/d*. For geometry with particular *D/d* value, six analyses were done by varying the *r/d* value. Table I gives the geometry configuration for parametric finite element study performed.

Table I Geometric configuration for parametric study (see Figure 2)

D/d	*r/d*					
1.1	0.05	0.10	0.15	0.20	0.25	0.30
1.2	0.05	0.10	0.15	0.20	0.25	0.30
1.5	0.05	0.10	0.15	0.20	0.25	0.30
2.0	0.05	0.10	0.15	0.20	0.25	0.30
2.5	0.05	0.10	0.15	0.20	0.25	0.30
3.0	0.05	0.10	0.15	0.20	0.25	0.30

Figure 6 shows a typical finite element model of the shaft. The element type used has ten nodes with three degree of freedom at each node: translation in the -x, -y, and –z direction. All nodes at the circular end were fixed and pure bending load was applied at the square cross section end.

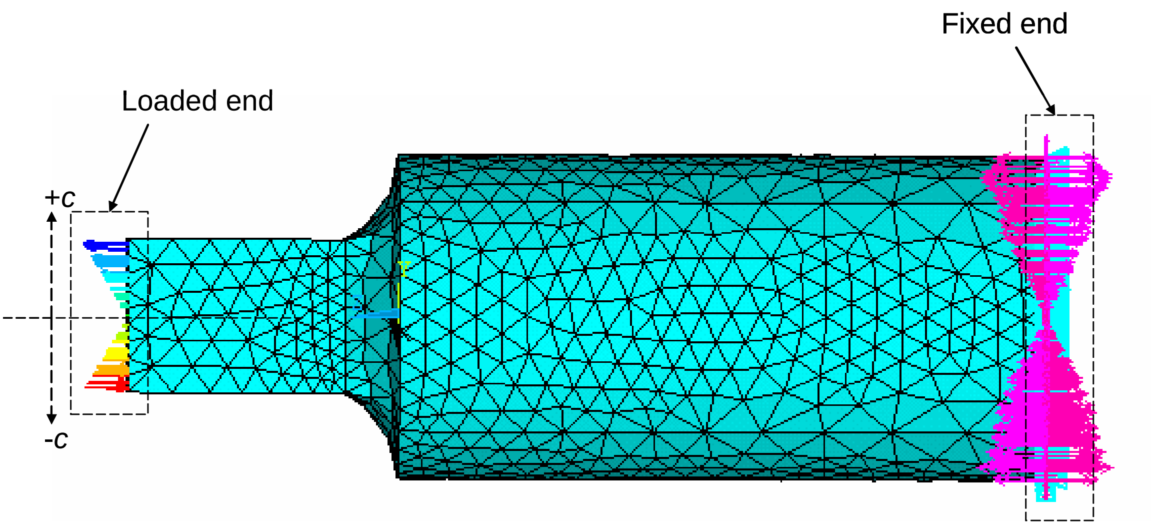

Figure 6 Typical finite element model of stepped shaft of circular to square cross-section

Figure 7 shows the normal bending stress distribution along the shaft due to pure bending. The maximum bending stress can be read from the graph. This maximum stress was then recorded for every combination of parameter *D/d* and *r/d* and divided by the nominal stress for pure bending [see equation (3)]. From this, a graph in line of Peterson's for uniform circular stepped shaft can then be plotted.

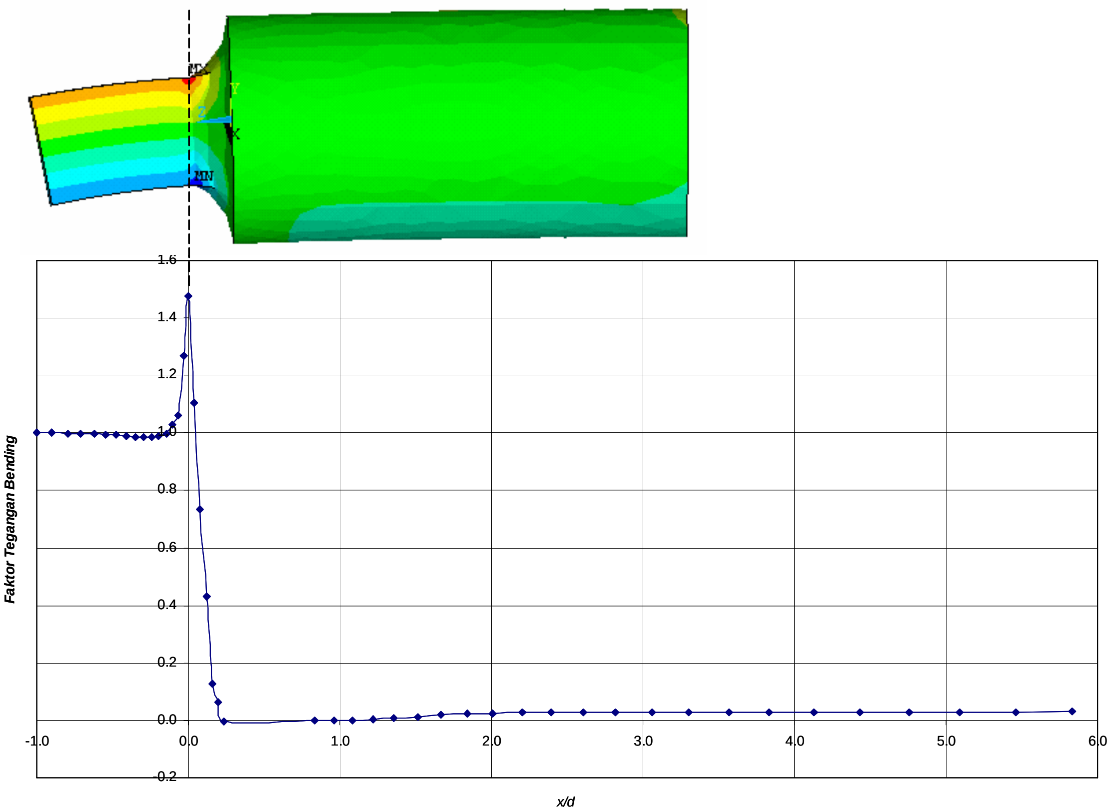

Figure 7 bending stress distribution in circular-to-square stepped-shaft due to pure bending

Results & Discussion

As can be seen from Figure 7, the maximum stress located at the region of change in cross-section. The stress plotted in Figure 7 is the von Mises stress. In fact, this stress is almost the same as the normal bending stress, as the load was only pure bending. A small different between von Mises and normal bending stresses results from the effect of Poisson's ratio. The maximum von Mises stresses were then recorded as shown in Table II:

Table II Maximum von Mises stress for varius geometry of stepped shaft under axial tension (MPa)

	r/d = 0.05	r/d = 0.10	r/d = 0.15	r/d = 0.20	r/d = 0.25	r/d = 0.30
R/d = 1.1	0.013856					
R/d = 1.2	0.014880	0.012416				
R/d = 1.5	0.015768	0.013144	0.011888	0.010688	0.01096	
R/d = 2.0	0.015208	0.013216	0.011592	0.011216	0.010872	0.010744
R/d = 2.5	0.016256	0.012864	0.011800	0.011128	0.010584	0.010768
R/d = 3.0	0.015704	0.013328	0.011600	0.011112	0.011160	0.010632

Stress concentration factor can be obtained from Table II by dividing the stresses by the nominal stress. Because the maximum stresses located at the shoulder, exactly at the corner of the square part, then the nominal stress can be obtained as:

$$\sigma_{nom} = \frac{M}{S} \tag{6}$$

Where M = applied pure bending, N.mm
S = section modulus of square-shaft, mm^3.

The section modulus S, of the square-shaft of diagonal d is given by:

$$S = \frac{bh^2}{6} = \frac{bh.h}{6} = \frac{A.h}{6} = \frac{A.d}{6\sqrt{2}} = \frac{d^3\sqrt{2}}{24} \tag{7}$$

In this analysis, the area of square-shaft was constant by setting the value of diagonal of the square constant, i.e. d = 50 mm, therefore:

$$S = \frac{50^3\sqrt{2}}{24} = 7365.7 mm^3$$

The applied moment bending M was 58.9 N.mm. From equation (6), the nominal bending stress is:

$$\sigma_{nom} = \frac{58.9 N.mm}{7365.7 mm^3} = 0.008 \text{ MPa.}$$

By dividing the maximum stresses in Table II with this nominal stress, stress concentration factors were obtained as shown in Table III. A graph of stress concentration factors can then be plotted from Table III as shown in Figure 8.

Table III: Stress concentration factor for various geometry of stepped shaft under axial tension

	r/d = 0.05	r/d = 0.10	r/d = 0.15	r/d = 0.20	r/d = 0.25	r/d = 0.30
R/d = 1.1	1.732					
R/d = 1.2	1.860	1.552				
R/d = 1.5	1.971	1.643	1.486	1.336	1.370	
R/d = 2.0	1.901	1.652	1.449	1.402	1.359	1.343
R/d = 2.5	2.032	1.608	1.475	1.391	1.323	1.346
R/d = 2.0	1.963	1.666	1.450	1.389	1.395	1.329

As can be seen from Figure 8, stress concentration factors decrease as the radius of filled to diagonal ratio increase. The graph shows a similar trend to the well-known Peterson's graph for stress-concentration factor for stepped shaft of uniform circular of square cross-section. By comparing Figure 8 with Figure 3 and 4, it seems that the effect of diameter to diagonal ratio, *D/d*, is not significant. It should be noted, however, that the spectrum of this ratio is small compared with Figure 3 and 4. A wide range of *D/d* values should be included and this is the subject of further publication elsewhere.

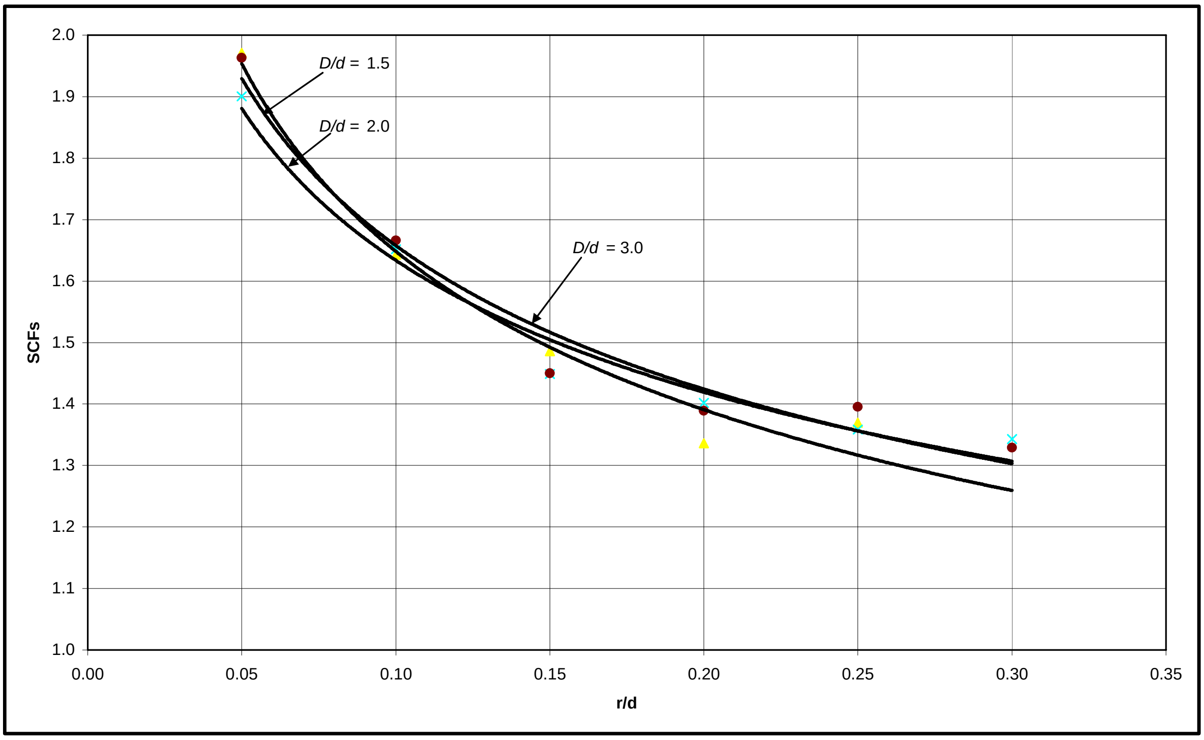

Figure 8 Stress concentration factor of stepped-shaft of circular-to-square cross section under pure bending

Conclusion

A graph for stress concentration factors of stepped shaft of circular to square cross section under pure bending load was produced. This graph was derived from parametric finite element study and can be directly used for design purpose. This graph shows a similar trend to the well known Peterson's graph for stress concentration factors of uniform circular or square stepped shaft.

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