Numerical Analysis of Buckling of Sandwich Plates with Aluminum Foam Core

N. T. Nam[a], I. S. Putra [a]*, T. Dirgantara[a], D. Widagdo[a], H. Homma[b]
[a]Lightweight Structures Research Group, Institut Teknologi Bandung
Jl. Ganesha 10, Bandung 40132, Indonesia
[b]Department of Mechanical Engineering, Toyohashi University of Technology
Tempaku-cho, 1-1 Hibarigaoka, Toyohashi 441-8580, Japan
*Corresponding contact: isp@aero.pauir.itb.ac.id

Abstract

This paper addresses the global buckling and wrinkling behaviors of sandwich plates with aluminum foam core and aluminum faces. The finite element eigenvalue analysis is conducted to predict buckling loads and buckling modes and to verify the analytical calculation. Good agreements between the numerical and analytical results are achieved. The results show that the aluminum foam sandwiches offer 2-10 times higher buckling load than common honeycomb sandwiches, while only 1-2 times heavier. Global buckling load decreases with aspect ratio in a similar way to homogenous plates, but increases with core thickness until wrinkling occurs.

Keywords: aluminum foam, sandwich plate, buckling, wrinkling, FEM

Introduction

In aeronautical, automobile and marine applications, lightweight structures are highly demanded since they provide weight reduction for higher payload. Sandwich structures are well-known for relatively high effective stiffness at very low weight. However, commonly used core materials, such as honeycomb and polymer foams, do not give sufficient rigidity, toughness and plasticity to many applications. Metal foams, especially aluminum foams, have been emerging as a novel class of materials with many excellent characteristics, such as high stiffness-to-density ratio, high energy-absorption capability, and good stability at high temperature [1].

Under compressive loads, sandwich structures may be damaged due to various causes in which buckling is the most serious because it usually occurs at low load and in operating condition. A sandwich plate may buckle in one of the three basic modes: *global buckling*, *symmetrical wrinkling*, and *anti-symmetrical wrinkling*. In global buckling mode, faces and core buckle together into a single buckle whose half wavelength is equal to an in-plane dimension. In wrinkling modes, faces deform locally with a large number of short wavelength buckles. When core does not exhibit any transverse deformation, wrinkling is anti-symmetrical; when it does equally toward the faces, wrinkling is symmetrical. Depending on complicated interactions between geometry parameters and materials properties, any of the basic modes may develop and usually be difficult to characterize. In practice, other failures may occur in conjunction with buckling, such as core crushing, face and core plastic deformation, and delamination. Since the 1940’s, many researches have been conducted to study the buckling behaviors of sandwich plates [see, e.g. 2-8], but most of the works concern sandwiches with weak core materials.

In this work, the buckling behaviors of sandwich plates with aluminum foam core are investigated in comparison with honeycomb sandwiches. Finite element method is mainly used for analysis, while analytical calculation is done as a benchmark. Sensitivity of buckling load and mode to aspect ratio and thickness is also considered.

Analytical calculation

The analytical methods introduced in this section are used for a fast prediction of critical load and mode. They are selected from a large number of theoretical works available in literature.

In an early analytical work to characterize global buckling, Seide and Stowell [2] developed a formula for simply-supported sandwich plates and columns, as given in Eq.1.

$$P_{cr,G} = \frac{D\pi^2}{b^2}k \tag{1a}$$

$$\text{with } k = \frac{\left(b/a + a/b\right)^2}{1 + \dfrac{\pi^2 D}{b^2 G_c t_c}\left(1 + b^2/a^2\right)} \tag{1b}$$

where $P_{cr,G}$ is the critical global buckling load; k is the critical global buckling load factor; D is the flexural rigidity per unit width; G_c is the shear modulus of the core; a and b are the length and width of sandwich plate, respectively; t_c is the thickness of core (see Fig.1). Because of its simplicity, the formula is still widely used in sandwich design and construction. The model is analogous to that of homogenous plates and involves the assumptions of anti-plane core (i.e. core deformation is dominant in transverse direction, still core shearing is considered) and plane-strain (i.e. either plate or column is treated to be infinitely wide). In an independent work, Hoff [3] also used this model except that local rigidity of the faces was taken into account. Using an energy approach to deal with various boundary conditions, he ended up with the following formula for the simply-supported plates and columns.

$$P_{cr,G} = \frac{1+\beta^2}{\beta}\left[(1+\beta^2)P_f + \frac{G_c t_c (1+\beta^2)P_0}{G_c t_c + (1+\beta^2)P_0}\right] \tag{2}$$

where $\beta = m(a/b)$ with m is the number of half-waves in loading direction (m = 1 for global buckling); $P_0 = D(\pi/b)^2$ with D defined as in Eq.1; $P_f = D_f(\pi/b)^2$ with D_f is the local rigidity of each face.

For wrinkling modes, Hoff and Mautner [4] modeled a sandwich plate as two elastic beams that represent faces deforming on an elastic foundation that represents core, and solve the problem for symmetrical wrinkling and anti-symmetrical wrinkling separately. The critical loads are calculated by Eq.3a-b for sandwiches with thick core and Eq.3c-d for those with thin core.

$$P_{cr,SW,thickcore} = 1.82 t_f \sqrt[3]{E_f E_c G_c} \tag{3a}$$

$$P_{cr,AW.thickcore} = 1.02 t_f \sqrt[3]{E_f E_c G_c} + 0.66 G_c t_c \tag{3b}$$

$$P_{cr,SW,thincore} = 1.634 t_f \sqrt{E_f E_c t_f / t_c} + 0.332 G_c t_c \tag{3c}$$

$$P_{cr,AW,thincore} = 1.08 t_f \sqrt{E_f E_c t_f / t_c} + 0.774 G_c t_c \tag{3d}$$

where E_f and E_c are the Young's modulus of face and core material, respectively; G_c is the shear modulus of core material; t_f and t_c are the thicknesses of face and core, respectively. The equations are very simple yet proved applicable for a wide range of sandwich plates and columns. However, the constants usually have to be modified semi-empirically for a particular application.

In unified models, global buckling and wrinkling are treated together, since global buckling is considered as a special mode of anti-symmetrical wrinkling. Displacement field is assumed, and critical load is calculated by establishing equilibrium equation [5] or minimizing first derivative of deformation energy [6]. Mathematically, finding critical load is equivalent to solving an eigenvalue problem that describes stability of the structure. Léotoing et al. [7] provided a close-form solution for the problem which is given as following.

$$P_{cr,SW} = \frac{\omega^2 E_f t_f^3 b}{6} + \frac{4E_c b}{\omega^2 t_f} + \frac{G_c t_c b}{3} \tag{4a}$$

$$P_{cr,AW} = \frac{\omega^2 E_f t_f b}{6}\left(t_f^2 + \frac{72 G_c E_c \left(t_c + t_f\right)^2}{\omega^4 E_f G_c t_f t_c^3 + 12\omega^2 E_f E_c t_f t_c + 24 E_f G_c}\right) \tag{4b}$$

where $\omega = m\pi/a$ with m is the number of half-waves in loading direction.

Finite element modeling

In this work, a large number of sandwich plates with aluminum foam core and aluminum faces are analyzed with finite element method using the Nastran commercial code. For comparison, honeycomb sandwiches are also analyzed.

Geometry and material properties

To observe a particular buckling mode, the geometry must be chosen accordingly. This can be done by making use of the analytical analysis. The geometry parameters of two typical plates are given in Fig.1 and Table.1. These parameters are also used for the honeycomb sandwiches. For the sensitivity studies, the width and the face thickness are fixed, whereas either the length or the core thickness is varied.

In the present models, materials of the core and faces are assumed to be homogenous, isotropic and elastic. A common aeronautical aluminum alloy with fixed values of properties is used for the faces, meanwhile Alporas is chosen as the core material. It is noted that properties of aluminum foams change with relative density, and relative density depends on overall dimensions and cell size [1]. For the specimens to be analyzed, variation in relative density versus overall dimensions is not significant, thus material properties of Alporas are assumed to stay constant. The material properties are given in Table.1. It is noteworthy that the total weights of the sandwich plates with Alporas core are only 1-2 times as high as those of sandwiches with honeycomb core, although the density of Alporas itself is 3-7 times higher than that of honeycomb. This is because the weights are mostly contributed by the faces made of aluminum alloy.

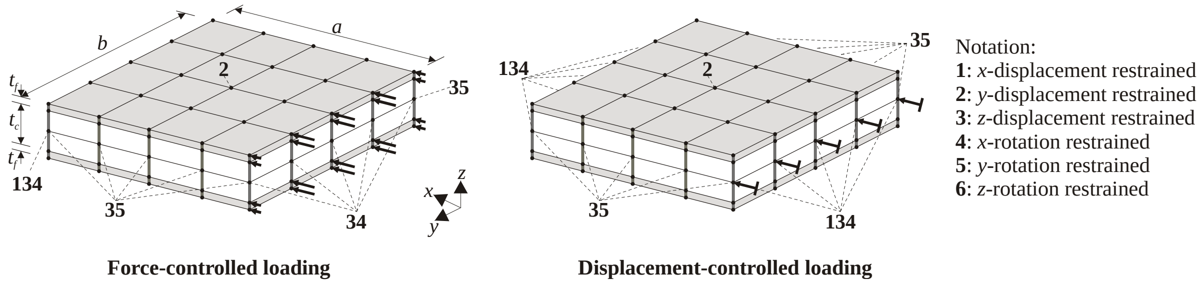

Figure 1: Illustration of geometry, boundary condition and loading. Length and width are denoted as *a* and *b*, respectively. Thicknesses of faces and core are denoted t_f and t_c, respectively.

Table 1: Geometry parameters and materials properties of the typical sandwich plates. The values given in bracket correspond to wrinkling specimens.

Geometry parameters		Material properties			
			Aluminum [9]	Alporas [10]	Honeycomb [8]
a (mm)	228 (228)	Young's modulus, E_x, E_y, E_z (MPa)	70000	1140 (1140)	0.01, 0.01, 298 (0.001, 0.001, 109) [a]
b (mm)	228 (228)	Shear modulus, G_{xy}, G_{yz}, G_{zx} (MPa)	-	-	0.01, 35.2, 60 (0.5, 15.5, 26.6) [a]
t_f (mm)	0.65 (0.65)	Poison's ratio, ν	0.33	0.33 (0.33)	0.25 (0.25) [a]
t_c (mm)	5 (25)	Density, ρ (g/cm^3)	2.8	0.22 (0.22)	0.064 (0.032) [a]
		Weight, (g)	189.2 [b]	57.2 (285.9)	16.6 (41.2)
		Total weight, (g)	-	246.4 (475.1)	205.8 (230.4)

[a] *Properties selected to avoid core crushing* [8]
[b] *Weight of both face sheets*

Elements and meshing

Hexagonal (8-node) solid elements are used to model the core and faces so that all possible deformations can be captured. Since face-core bonding is assumed to be perfect and adhesive layers are ignored, face and core are connected by merging identically positioned nodes that belong to both face and core elements. In a common experimental set-up for simply-supported plates, every edge is hold by a series of short rollers in order that any local part of the edge simply rotates about its axis (the rotation may differ from part to part, i.e. from roller to roller). In this work, the roller is successfully modeled as rigid line element that connects dependent nodes to an independent node. Selected displacements and rotations of the dependent nodes are forced to be consistent with those of the independent node following pre-defined mathematical constraints.

To achieve correct results, dense and even meshing is applied. A number of 40-100 elements are meshed in *x*- and *y*-direction; in z-direction, the faces are meshed into 1-2 elements and the core is meshed into 2-8 elements. The total number of elements is between 7000 and 120000, where the specific number is determined by convergence study. In element meshing, care should be taken to keep solid elements from ill-conditions, such as too high aspect ratio.

Boundary condition and loading

For the sandwich plates analyzed in this work, their four edges are simply-supported, i.e. transverse displacements (in *z*-direction) are restrained at the edges. The constraints are only imposed on the independent nodes, but the dependent nodes are also constrained properly due to the rigid line elements. The restraint “2” shown in Fig.1 is to keep the structure from rigid-body translation in *y*-direction that causes stiffness matrix over-determined.

Either *force-controlled* or *displacement-controlled* loading is imposed. In the former scheme, unit nodal forces are applied to the faces, since sandwich faces carry most of external loads. It is noted that the nodes at corners are loaded only by a half of unit force so that stress distribution around the loaded edge is uniform. In the latter scheme, unit nodal displacements are applied to the independent nodes of the rigid elements. Since buckling load has a value of force, displacement load is transformed into force via geometry and materials properties. However, experience showed that there should be no significant difference in results of the two schemes.

Buckling load computation

To calculate critical buckling load, the linear eigenvalue analysis is used, where the critical load is computed as compressive load corresponding to the lowest solution of an eigenvalue problem and buckling mode shape is calculated from the corresponding eigenvector. Although the critical load is exact, displacement field (thus stress field) of a buckled plate obtained from the eigenvalue analysis does not have exact magnitude but is scaled by an arbitrary number.

Results and discussion

Comparison with honeycomb sandwiches

The results of global buckling and wrinkling loads computed for the typical sandwich plates with Alporas and honeycomb core are given in Table.2.

Table 2: Buckling load calculated for sandwich plates with Alporas core and honeycomb core

	Global buckling load (N/mm)	Wrinkling load (N/mm)
Alporas sandwich		
Present FEM result	**609**	**6111**
Analytical result calculated from Eq.1	541	7999
Analytical result calculated from Eq.2	544	7996
Analytical result calculated from Eq.4	528	6118
Honeycomb sandwich		
Present FEM result	**272**	**557**
Analytical result calculated from Eq.3	-	697
Analytical result given in [6]	298	497
Experimental result given in [8]	234	361

It can be seen that the global buckling load of the Alporas sandwich is 2 times higher than that of the honeycomb sandwich, while their total weight are almost the same (see Table.1). It is because Alporas is isotropic and has a much higher axial stiffness of than does the orthotropic honeycomb. The advantage of Alporas over honeycomb is more significant in its wrinkling load, which is about 10 times higher than that of honeycomb, while the Alporas sandwich is only twice as heavy as the honeycomb sandwich. The superiority in wrinkling resistance of Alporas can be explained by its higher shear stiffness that keep sandwich faces cooperating better. Both of the advantages are not surprising because aluminum foams are metal-based and isotropic materials while honeycombs are usually polymer-based and always orthotropic.

In comparison with the analytical results, the present results show a good agreement with the maximum discrepancy of 30%. The main reason of discrepancy is that all of the analytical methods are based on one- or two-dimensional models with assumptions that overestimate or underestimate many important parameters and factors. While the global buckling load calculated for the honeycomb sandwich agrees very well with the experiment of Pearce and Webber [8], the wrinkling load differs by 35% from the experimental value. As reported by the referred author, the experiment failed to capture wrinkling behavior, since core crushing had occurred first; thus the value provided is indeed not corresponding to a wrinkling load.

Effect of aspect ratio

In this sensitivity analysis, the global buckling specimen is chosen rather than the wrinkling specimen, because global buckling is more sensitive to in-plane dimensions than wrinkling (according to the analytical analysis).

As shown in Fig.2, the numerical results correlate very well with the analytical ones. Therefore, the classical and unified theories with buckling load formulae given in Eq.1, Eq.2 and Eq.4, are verified.

For small aspect ratio, the buckling load is very high, because the effectively large width tends to strengthen the sandwich plate. As the aspect ratio increases until it reaches unity, global buckling load decreases drastically. Regardless of any further increase in aspect ratio beyond the value of unity, buckling load stays almost at a constant value. It is interesting that this global buckling behavior of the aluminum foam sandwiches is analogous to buckling of homogenous plates. That means, the aluminum foam core offers a very good connection to the faces of thin sandwich plates.

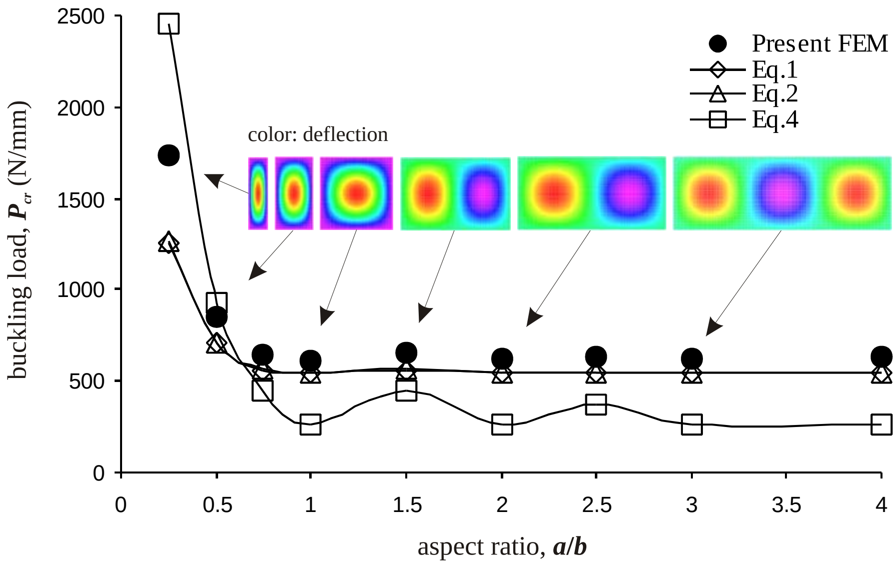

Figure 2: Effect of aspect ratio to buckling aluminum foam sandwich. The results are for the typical global buckling specimen with only the length *a* varying.

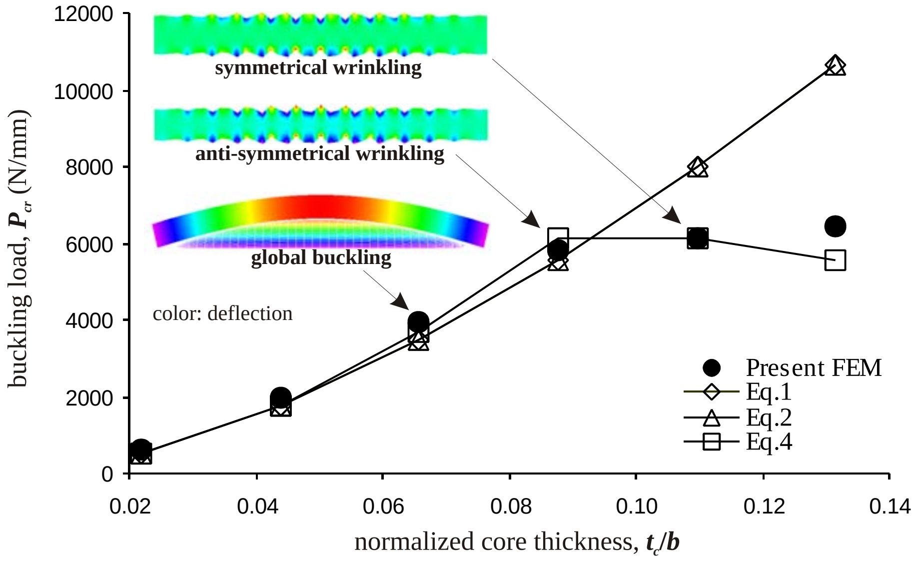

Figure 3: Effect of core thickness to buckling of aluminum foam sandwich. The results are for any of the typical specimens with only the core thickness t_c varying.

Effect of core thickness

In this investigation, the core thickness t_c of the typical specimens is varied from 5 mm to 30 mm and normalized with respect to the width *b*, while all other geometry parameters are fixed.

As can be seen in Fig.3, core thickness affects both buckling load and buckling shape. At the beginning stage of increasing core thickness, the sandwich plate exhibits global buckling mode which is considered as anti-symmetrical wrinkling with only one half-wave, and corresponding buckling load increases with core thickness which is also analogous to homogenous plates. At a certain value of the core thickness, the buckling load approaches an onset, and the buckling mode becomes truly anti-symmetrical wrinkling with a large number of half-waves. It is expected that several other truly anti-symmetrical wrinkling modes with a less number of half-waves should occur within certain range *before* that onset. For further increase of core thickness after the onset, the wrinkling mode shape changes from anti-symmetrical to symmetrical, and buckling load slightly increases for a short range of core thickness. It is expected that the buckling load decreases with still further increase of core thickness, because of less cooperation between sandwich faces regardless of core material amount added. This behavior was well explained for sandwich columns by the unified theory of Hadi and Matthews [6] and Léotoing [7].

The finite element results are consistent to the analytical ones, especially to that of Léotoing [7] which is given in Eq.4. The other two analytical methods can only predict global buckling (as introduced before), and thus have given the unsafe predictions for wrinkling.

Conclusions

The present finite element analysis of buckling of sandwich plates with aluminum foam core gives the results consistent to the analytical results. It is shown that aluminum foam, when used as core of sandwich plates, offers 2-10 times higher buckling resistance than honeycomb does. The sensitivity study on aspect ratio shows that buckling behavior of aluminum foam sandwich plates is similar to the behavior of homogenous plates. As core thickness increases, buckling load increases until an onset value and buckling mode tends to change from global buckling to wrinkling.

Acknowledgement

The financial support of the AUN/SEED-Net program is gratefully acknowledged.

References

[1] Ashby et al., 2000, *Metal Foam: a Design Guide*, Elsevier Science

[2] Seide and Stowell, 1948, Elastic and plastic buckling of simply supported solid-core sandwich plates in compression, *NACA TN*, 967

[3] Hoff, 1950, Bending and buckling of rectangular sandwich plates, *NACA TN*, 2225

[4] Hoff and Mautner, 1945, The buckling of sandwich-type panels, *J. Aero. Sci.*, 12, 285-297

[5] Benson and Meyers, 1967, General instability and face wrinkling of sandwich plates – unified theory and applications, *AIAA J.*, 5 (4), 729-739

[6] Hadi and Matthews, 2000, Development of Benson-Meyers theory on the wrinkling of anisotropic sandwich panels, *Comp. Struct.*, 49, 425-434

[7] Léotoing et al., 2002, First applications of a novel unified model for global and local buckling of sandwich columns, *Euro. J. Mech. A/Solids*, 2 (4), 683-701

[8] Pearce and Webber, 1973, Experimental buckling loads of sandwich panels with carbon fibre faceplates, *Aero. Quarterly*, 295-312

[9] ASM, 1998, *Metal Handbook*, Vol.2, Ed.5

[10] Andrews et al., 1999, Compressive and tensile behaviors of aluminum foams, *Mat. Sci. Eng.*, A270, 113-124