SOLUTION FOR THE FLUID FLOW DYNAMICS OF AN AXISYMMETRIC JET WITH SWIRL AND ITS IMPLEMENTATION ON FAN FLOWS

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ABSTRACT

In this preliminary study, the fluid flow dynamics caused by an axisymmetric jet with swirl had been investigated. Similarity method was used to derive a system of ordinary, nonlinear differential equations governing the flow pattern of 3-dimensional, free turbulent axisymmetric jet with swirl, valid in a relatively restricted domain in the vacinity of the main flow. Gaussian distribution was assumed and the system of equations was integrated numerically using Runge-Kutta method with shooting approach. At the same time, a simple measurement employing a micro-manometer was conducted to investigate the axial velocity distribution to determine the typical values of parameters for the free turbulence flow of a small fan. The preliminary results for the dynamic behavior of pressure, the axial, tangential as well as radial velocity components and the effect of swirling were reported. The experiment results, based on this theory, matched well with previous prestigious investigation.

INTRODUCTION

A fast number of industrial products had been manufactured in developing countries such as Indonesia. Among of them were propeller typed products such as fans, blowers, ship and boat propellers. Their important roles in supporting the fast growing industries in Indonesia needed to be reconsidered more carefully, especially their needs for more thoroughful research.

Flow caused by propeller typed devices could be considered as one type of axisymmetrical jet flows with swirl. The dynamics of the flow from blowers were very important information for combustion chamber designer, while that from ship propeller was an important consideration for determining the proper choice of the propeller type.

A lot of information on jet characterization was available in textbooks, but very limited mathematical investigation was available for axisymmetric jet with swirl. Two dimensional laminar jets (Schlichting, 1979, White, 1974 and Batchelor, 1967) and circular laminar jets (Schlichting, 1979 and Batchelor, 1967) had been solved by Schlichting using a similarity technique employing stream function type formulation.

Two dimensional turbulent jets (Schlichting, 1979 and White, 1974) and circular turbulent jets (Schlichting, 1979 and White, 1974) had also been investigated by Tollmien using Prandtl's mixing length hypothesis. According to Prandtl, the increase in width of the two dimensional and circular turbulent jets was proportional to the distance from the origin of the jets. The flow character of submerged jet had been derived by Landau (Batchelor, 1967 and Landau and Lifshitz, 1987) using

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Stokes stream function defined in a spherical polar coordinate. The result showed that as the fluid jet moved rapidly away from the source, it induced slow-moving fluid entraining from outside the jet.

Related studies such as swirling typed-motion had also been conducted. Flow near laminar rotating disk (Schlichting, 1979, White, 1974, Batchelor, 1967, and Landau and Lifshitz, 1987) had been elucidated by Von Karman. Fluid motion near a stationary wall when the fluid at large distance above it rotated at constant angular velocity had been analyzed by Boedewadt (Schlichting, 1979). These analyses employed similarity method which gave a system of ordinary, nonlinear differential equations to be solved numerically.

In this study the dynamics of the fluid flow caused by an axisymmetric jet with swirl was investigated. Similarity method was used to derived a system of ordinary, nonlinear differential equations from 3-dimensional, free turbulent governing equations for an axisymmetric jet with swirl, valid in a quite restricted domain in the vacinity of the main flow. A Gaussian distribution was assumed and the system of equations was integrated numerically using Runge-Kutta method with shooting approach. A simple measurement employing a micro-manometer was conducted to investigate the axial velocity distribution to determine the typical values of parameters for the free turbulence flow of a small fan.

Further investigation utilizing a 2-dimensional DANTEC hotwire anemometer is currently in progress to measure the jet flow turbulence as well as its spectral characteristics.

AXISYMMETRIC JET FLOW MODELING

A system of governing equations for an axisymmetric jet flow with swirl could be presented as follows

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0\ ;\qquad u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\varepsilon_T\left\{\frac{\partial^2 w}{\partial r^2}+\frac{\partial w}{r.\partial r}+\frac{\partial^2 w}{\partial z^2}\right\} \tag{1a,b}$$

$$u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}=\varepsilon_T\left\{\frac{\partial^2 v}{\partial r^2}+\frac{\partial (v/r)}{\partial r}+\frac{\partial^2 v}{\partial z^2}\right\};\qquad u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=\frac{-1}{\rho}\frac{\partial p}{\partial r}+\varepsilon_T\left\{\frac{\partial^2 u}{\partial r^2}+\frac{\partial (u/r)}{\partial r}+\frac{\partial^2 u}{\partial z^2}\right\} \tag{1c,d}$$

which consisted of the equation of continuity and three components of Navier-Stokes equations governing the momentum balance in axial, tangential and radial directions. In this problem the components of velocity and distance in radial, tangential and axial directions were stated as (u,v,w) and (r,θ,z) respectively. In this axisymetrical problem. the tangential derivative was zero. The boundary conditions of the problem were then described by

$$z\to 0\,;u=0,v=\alpha\omega r.\exp(-r^2/2c^2z^2)\,;\qquad w=\beta\sqrt{\varepsilon_T}\,\omega\exp(-r^2/2c^2z^2) \tag{2a}$$

$$z\to\infty;u=0,v=0 \tag{2b}$$

The coefficients α, β, and c were constant. This boundary condition was set up that at z→0, the radial velocity weakened rapidly farther away from the main flow. This jet flow was turbulent, and in the above equations the free turbulent type flow was characterized by parameter ε_T, namely virtual kinematic viscosity,

$$\frac{\varepsilon_T}{\sqrt{K}} = 0.0161 \text{ and } K = \int_0^\infty u^2 r dr$$

(3a,b)

In order to get the pattern of this axisymmetric jet flow dynamics, the similarity method was employed using the following similarity variables

$$\zeta = z\sqrt{\omega/\varepsilon_T}\ ; \qquad u = r\omega F(\zeta)\exp\left(-r^2/2c^2z^2\right) \quad ; \quad v = r\omega G(\zeta)\exp\left(-r^2/2c^2z^2\right)$$

(4a,b,c)

$$p = \rho\, \varepsilon_T\, \omega P(\zeta)\exp\left(-r^2/2c^2z^2\right); \quad w = \sqrt{\varepsilon_T \omega} H(\zeta)\exp\left(-r^2/2c^2z^2\right)$$

(4d,e)

In this preliminary effort, a relatively simple formulation was preferred. A small, aspect-ratio like parameter, $\in$, was defined, as follows

$$\in = \frac{r}{cz}; \qquad 0 < \in << 1$$

(5)

Therefore this investigation would be focussed in a relatively restricted domain in the vacinity of the main axis, $0 < r << cz$. Using the similarity variables (4a,b,c,d,e) and the equation (5), the system of equations (1a,b,c and d) could be presented as

$$2F\left(1-\tfrac{1}{2}\in^2\right) + H'\left(1+\in^2 \tfrac{H}{\zeta H'}\right) = 0$$

(6a)

$$-\in^2 FH e^{-\in^2/2} + HH'\left(1+\in^2 \tfrac{H}{\zeta H'}\right)e^{-\in^2/2} = P' + \in^2 \tfrac{P}{\zeta} + \tfrac{H}{c^2\zeta^2}\left(\in^2 - 2\right) + H'' + \in^2\left(2\tfrac{H'}{\zeta} - 3\tfrac{H}{\zeta^2}\right) + \in^4 \tfrac{H}{\zeta^2} \quad (6b)$$

$$FG(2-\in^2)\, e^{-\in^2/2} + HG'\left(1+\in^2 \tfrac{G}{\zeta G'}\right)e^{-\in^2/2} = + \tfrac{G}{c^2\zeta^2}\left(\in^2 - 4\right) + G'' + \in^2\left(2\tfrac{G'}{\zeta} - 3\tfrac{G}{\zeta^2}\right) + \in^4 \tfrac{G}{\zeta^2}$$

(6c)

$$F^2(1-\in^2)e^{-\in^2/2} + F'H\left(1+\in^2 \tfrac{F}{\zeta F'}\right)e^{-\in^2/2} - G^2 e^{-\in^2/2} = + \tfrac{F}{c^2\zeta^2}\left(\in^2 - 4\right) + F'' + \in^2\left(2\tfrac{F'}{\zeta} - 3\tfrac{F}{\zeta^2}\right) + \in^4 \tfrac{F}{\zeta^2}$$

(6d)

One could find the system of basic governing equations for this axisymmetric jet, valid on the asymptotic limit $\in \to 0$, from equations (6a-d) in the form of a system of nonlinear ordinary differential equations. In the following, the system was presented sequentially from the equation of continuity followed by the equation for momentum balance in radial, tangential and axial directions.

$$2F + H' = 0; \qquad F^2 + F'H - G^2 = -4\tfrac{F}{c^2\zeta^2} + F'' + \tfrac{P}{c^2\zeta^2}$$

(7a,b)

$$2FG + HG' = -4\tfrac{G}{c^2\zeta^2} + G'' \quad ; \quad HH' = -P' - 2\tfrac{H}{c^2\zeta^2} + H''$$

(7c,d)

With some modification, the following system of equations would be found

$$H' = -2F \quad ; \quad F'' = F^2 + F'H - G^2 + 4\frac{F}{c^2\zeta^2} - \frac{P}{c^2\zeta^2} \qquad (8a,b)$$

$$G'' = 2FG + HG' + 4\frac{G}{c^2\zeta^2} \quad ; \quad P' = 2FH - 2FH' - 2\frac{H}{c^2\zeta^2} \qquad (8c,d)$$

The boundary conditions (2a,b) could be written as

$$\zeta \to 0\,;\, F \to 0, G \to \alpha\,, P \to 0, H \to \beta\,;\quad \zeta \to \infty; F, G \to 0 \qquad (9a,b)$$

The system of equations (8a-d) and (9a,b) was highly nonlinear which possibly could only be solved numerically. The followings would be discussed the first attempt to solve this system using Runge Kutta method based on shooting algorithm.

THE CHARACTERISTIC OF GAUSSIAN DISTRIBUTION

In the boundary conditions (2a,b) and the similarity variables (4a-e), Gaussian velocity and pressure distributions were assumed, partly following the conclusion of Reichardt's experiment (Schlichting, 1979). These Gaussian distributions demonstrated the following behaviors that were very helpful to identify and measure some turbulent characteristic parameters. In this study, the distribution of tangential velocity component could be expressed as $v = r\omega G(\zeta)\exp(-r^2/2c^2z^2)$. It meant that for a certain value of z one could find the location of its maximum values by insisting $(\partial v/\partial r) = 0$, with $G(\zeta) = O(1)$, which was at $r = \pm\, cz$, with $v_{max} = \omega cz\, G\{z\sqrt{(\omega/\varepsilon_T)}\}e^{-1/2}$. It was important to note that $G(\zeta)$ decayed down rapidly with the increase of ζ. Meanwhile, the axial velocity, $w = \sqrt{\varepsilon_T \omega}\, H(\zeta)\exp(-r^2/2c^2z^2)$ which had Gaussian pattern, at the location when $v = v_{max}$, then $w = 0{,}6065\ w_{max}$. These relationship would be used to extract some parameters such as the turbulent jet spreading constant c, the half width at half depth $b_{1/2}$, and the virtual kinematic viscosity ε_T such as presented in Table 1.

FAN FLOW PATTERN MEASUREMENT

In this preliminary investigation, a three-bladed 50 Hz, 220V-25W fan was hung 0.46 m under the laboratory ceiling of 3.42 m height. The blade operated at 223 rpm and had 0.71 m diameter and each blade had 182 cm^2 area with 4.67 aspect ratio. The fan velocity distribution was measured with a simple method utilizing Zephyr digital micro-manometer from Solomat equipped with a Pitot tube. A 40 cm length, pointed cone was fitted to the front end of the fan axis to reduce the recirculation flow. In order to visualize as well as to measure the pattern of the flow direction in a certain plane at some distance away from the fan, a tuft grid method was used (Nakayama, 1988). A grid of wires, of 0.6 mm diameter arranged with 10 cm distance between the wires, was located at certain distance of 100 cm, 110 cm and 150 cm from the fan. Twelve centimeter sewing threads were fastened at some cross sections of the wire to show the flow direction. This type of thread was relatively sensitive to follow the direction of the flow since it had a very rough fur surface, relatively soft and light weight.

By measuring the direction of the flow (angle θ and ϕ as illustrated in Fig. 2. as shown by the direction of the thread, and also by measuring the flow velocty V with the digital micromanometer-Pitot tube, one could calculate the axial component w of the fan flow velocity at any distance using the following formulae

$$w = V \frac{\sin\theta \tan\Phi}{\sqrt{\sin^2\theta + \tan^2\Phi}} \qquad (10)$$

The results were presented at Fig.3. showing the axial velocity distributions a) at z_l = 100 cm, b) at 110 cm and c) at 150 cm away from the fan position. Curved lines marked with (♦) showed results measured using digital micro-manometer, and curves without mark showed the axial velocity distribution predicted based on Gaussian distribution.

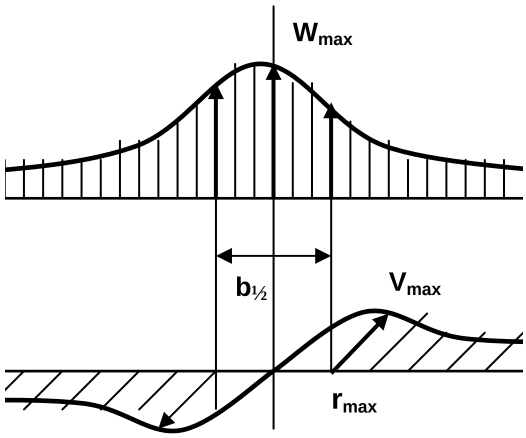

Fig.1 The characteristics of a Gaussian velocity distribution.

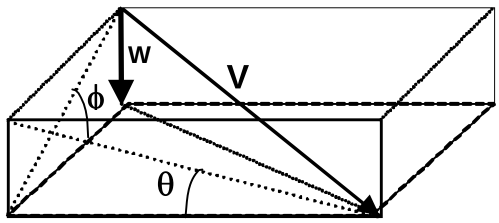

Fig.2 Determination of the axial velocity component, w, when the flow velocity V and the angles θ and Φ were known.

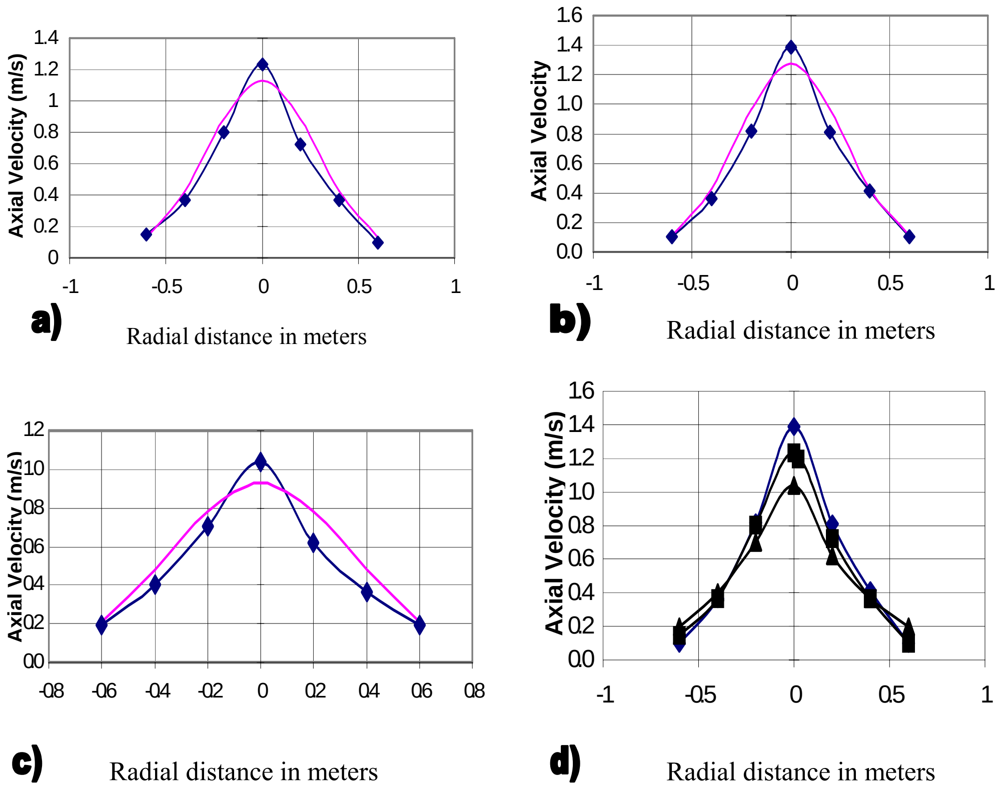

Fig. 3. Axial velocity distributions in radial direction a) at z_l = 100 cm, b) at 110 cm and c) at 150 cm distance from the fan position. Curved lines marked with (♦) show results measured using micro-manometer, and curves without mark show the axial velocity distribution predicted based on Gaussian distribution. Combination of these velocities is presented at d).

Table 1. Calculation of c based on Gaussian velocity distribution and calculation of ε_T based on Reichardt theory.

No.	Z_L	r_{max}	C
1.	100 cm	0.182 m	$c_{1-2} = 0.070$
2.	110 cm	0.189 m	$c_{1-3} = 0.112$
3.	150 cm	0.238 m	$c_{2-3} = 0.122$
Averaged			$c_{ave} = 0.101$

No.	Z_L	$b_{1/2}$	W_{max}	$\varepsilon_T \times 10^2$
1.	100 cm	0.250 m	1.390 m/s	0.8896 m^2/s
2.	110 cm	0.257 m	1.233 m/s	0.8113 m^2/s
3.	150 cm	0.321 m	1.038 m/s	0.8530 m^2/s
Averaged				0.8513 m^2/s

Table 1 indicated that for this fan the averaged value of c = 0.101 very close to the result measured by Reichardt c = 0.0848 (Schlichting, 1979). The values of virtual kinematic viscosity ε_T was calculated employing Reichardt's formula for free turbulent circular jet

$$\varepsilon_T = 0.0256 \; b_{1/2} \; W_{max} \quad (11)$$

As soon as parameters for the turbulent axisymmetric jet had been found, it was expected that one could figure out the remaining pattern of the flow dynamics of the jet by solving the system of nonlinear differential equations (8a-d) and (9a,b) that demonstrated its similar behavior when the flow was Gaussian in character.

NUMERICAL SOLUTION

In this primarily study, equations (8a-d) and (9a, b) were solved using numerical method employing Runge-Kutta subroutine. It was worth to mention that to solve this system of equations required hard effort due to the fact that this system of equations was highly nonlinear and in the form of a boundary value problem such that in applying Runge Kutta subroutine appropriately, shooting method had to be adopted. Numerical computation was conducted using BVPMS subroutine from Digital Visual Fortran Software Version 5.0. It was important to note that computation was not always convergent. When it was convergent the result had to be checked to get satisfying results.

Some preliminary results were presented on Fig. 4 showing the distribution of axial, tangential and radial velocity components, in the form of similarity functions H, G, F and pressure P, with respect to similarity variable $\zeta = z\sqrt{(\omega/\varepsilon T)}$. In this case the boundary condition for $\zeta \rightarrow 0$ was applied, where $H(0) = \beta = 10.$, and the turbulent spreading parameter c = 0.380. The result showed 3 cases of different source swirling velocity parameters, $G(0) = \alpha$, where for a) $\alpha = 0.50$, b) $\alpha = 0.75$ and c) $\alpha = 1.00$.

One should note that the effect of swirling was significant, the stronger the swirling parameter was introduced, the farther the radial velocity component propagated away. The swirling amplification gave lesser significant effect to the pressure and tangential velocity variations, and the least effect was to the change of the axial velocity distribution.

Further investigation was being conducted to characterized the flow dynamics of the fan and blower utilizing Dantec 2-dimentional hotwire anemometer.

CONCLUSIONS

In this study, the mathematical modeling had been formulated for a 3-dimensional , free turbulent, axisymmetric jet flow with a swirl. This model was used to characterize the fluid flow dynamics from a fan. A system of highly nonlinear differential equations was found to explain the Gaussian behavior and similar-type of the swirling jet flow.

In order to obtain typical flow parameter of the fan, a simple experiment had been conducted. Typical values of spreading parameter c = 0.101 and virtual kinematic viscosity $\varepsilon_T = 0.851\ m^2/s$ were found for this small fan. This virtual viscosity was much higher than the apparent viscosity of the fluid.

Numerical method had been adopted to solve the nonlinear system in order to get the similar behavior of the axial, tangential and radial velocity components. It was found that the swirling of the flow gave significant effects to the distribution of radial velocity component. The stronger the swirling was introduced to the flow the farther the effect suffered by the radial component. The effect was weaker to the pressure and tangential velocity variations, and the least was to the axial velocity distribution.

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NOTATION

r, z	radial, axial distance (m)	V	resultant velocity (m/s)
u,v,w	radial, tangential and axial velocity components (m/s)	θ, Φ	azimuthal angles in x- and y-directions (radian)
p	pressure (Pa)	v_{max}, w_{max}	tangential and axial maximum velocities (m/s)
ρ	density (kg/m^3)	r_{max}	radial distance of the maximum tangential velocities (m)
ε_T	virtual kinematic viscosity (m^2/s)	$b_{1/2}$	half distance of half depth (m)
ω	swirl angular velocity (rad/s)	α, β	constant
c	turbulence spreading parameter	ζ	similarity variable
F,G,H,P	dimensionless similarity functions for radial, tangential and axial velocities		
$\in$	aspect ratio-like parameter		

REFERENCES

Batchelor, G.K. (1967) *An Introduction to Fluid Dynamics*, Cambridge University Press, pp.
Landau, L.D. and Lifshitz, E.M. (1987) *Fluid Mechanics*, Pergamon Press, New York, pp.
Nakayama, Y. (1988) *Visualized Flow,* Pergamon Press, New York, pp.
Schlichting, H. (1979) *Boundary Layer Theory*, 7th ed., McGraw-Hill, New York, pp.
White, F.M. (1974) *Viscous Fluid Flow*, McGraw-Hill, New York, pp.