M6-006 Slip Boundary Condition of Fluid Flow: a Review

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ABSTRACT

Reynolds lubrication theory assumes that there is no-slip on the interfaces between the lubricant and the contacting surfaces. During the last decades, however, it is found that the wall slip often occurs. This paper presents the review on existing work concerning lubrication theory with slip from a theoretical and numerical as well as from an experimental point of view. A failure of the classical no-slip boundary condition is now suggested by a series of researches, especially at microscopic scale. The evidence of the wall slip is a challenging problem in lubrication mechanic and has also potential practical consequences in many areas of engineering and applied science where liquids interact with small scale system. In the future, the lubrication model with boundary slip conditions (Reynolds equation with slip) will be derived appropriately by including several factors.

Keywords: Reynolds equation, MEMS, wall slip

1. Introduction

In the classical fluid mechanic, it is general practice to use the Reynolds equation, to describe the flow in the gap between the surfaces of the contacting machine elements. The derivation of the Reynolds equation is usually based on the assumption of 'no-slip' between the lubricant and the contacting surfaces, i.e. the lubricant velocities at the surfaces are set equal to the surface velocities. However, this assumption may be no longer be accurate, especially at microscopic scale. During the last decades, the evidence of the wall slip at the liquid-solid interface has been reported by many researchers. The boundary wall slip is a complicated phenomenon related to many different factors such as the surfaces wettability, shear rate, viscosity of the fluid, surface roughness, and so on and is not well understood yet.

In micro- or nanoscopic scales such as MEMS (Micro-Electro-Mechanical-System), the boundary condition will play a very important role in determining the fluid flow behavior. Control of the boundary condition will allow a degree of control over the hydrodynamic pressure in confined systems and be important in a lubricated-MEMS.

This paper describes the review on existing work concerning with boundary slip condition. First the limitation of the validity of Reynolds equation will be discussed, followed by a brief introduction to slip boundary condition in general. The review on the evidences of the wall slip at the interfaces will be discussed.

2. Classical Reynolds Equation

When modeling elastohydrodynamic lubrication (EHL), the Reynolds equation is the main partial differential equation to be applied. The Reynolds equation is a differential equation describing two-dimensional pressure distribution in a thin film formed between two moving surfaces [1]. The Reynolds equation is essentially a continuity equation. Before applying the Reynolds equation, it will be derived by considering the viscous shear forces and pressure forces acting on a small element in the fluid film between the two separated surfaces (Fig. 1).

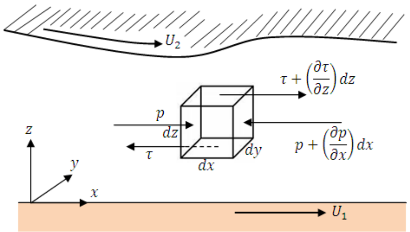

FIGURE 1: *Equilibrium of viscous shear forces and pressure forces, reproduced from* [2].

As shown in Fig. 1 two surfaces are separated by a thin film. The *x*- and *y*- axes are oriented along surface 1 and the normal to this surface being the *z*-axis. The equilibrium of forces acting on the element with sides *dx*, *dy*, and *dz* is considered. In deriving Reynolds equation, the following assumptions are made after Reynolds [1]:

a. The flow is laminar;
b. The fluid is Newtonian and the coefficient of viscosity is constant;
c. Fluid pressure does not change across the film thickness, because the film is very thin;
d. There is no-slip between the fluid and the solid surface;
e. Compressibility of the fluid is negligible. It means that using thin-film approach, assuming constant pressure and temperature across the film thickness, also implies a constant density across the film thickness.

With these assumptions, hence the two-dimensional Reynolds equation is given by:

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial y}\right)=\frac{U_1+U_2}{2}\frac{\partial \rho h}{\partial x}+\frac{V_1+V_2}{2}\frac{\partial \rho h}{\partial y}+$$
$$\frac{\rho h}{2}\frac{\partial}{\partial x}(U_1+U_2)+\frac{\rho h}{2}\frac{\partial}{\partial y}(V_1+V_2)+\frac{\partial \rho h}{\partial t} \quad (1)$$

where U_1, V_1 is the velocity of the lower surface in the *x*- and *y*-direction, respectively, and the U_2, V_2 is the velocity of the upper surface for the same direction. In the left term the change in flow in *x*- and *y*-direction is given, initiated by the terms on the right side, i.e. the wedge effect ($\partial h/\partial x$), the stretch effect ($\partial U/\partial x$), and the squeeze effect ($\partial h/\partial t$). The wedge effect is a result of a sliding motion forcing the fluid in a converging wedge shaped region, while the stretch effect is a result of elongation of the surface, and the squeeze term is a result of pressure generation due to the variation of surface gap (film thickness) [2].

3. Limitation of the Validity of the Reynolds equation

An important aspect of lubrication with very thin rough films is the possible breakdown of the classical Reynolds equation. A common assumption in the derivation of the Reynolds equation is that the scale along the fluid film is three orders of magnitude larger than the scale across the fluid film. When the derivation is carried out, inertia is neglected due to the small Reynolds number in combination with the thin film in the contact region.

The limitation of the validity of Reynolds equation related to the film aspect ratio can be referred to as a geometric limitation. Some of the earliest examples are the works of Sun and Chen [3], Phan-Thien [4] and Myllerup and Hamrock [5, 6]. They studied about this limitation in relation to the effect of surface roughness. Most of these studies are analytical. So far, only a few authors such as Odyck and Venner [7, 8], and Sahlin *et al.* [9] have solved numerically the Reynolds and Stokes equations for tribologically relevant problems.

Odyck and Venner [7] discussed the accuracy of the predictions of the Reynolds model in relation to the local geometry of the gap. They investigated the differences between Stokes (Navier-Stokes equations without inertia terms) and Reynolds equations under isoviscous, Newtonian and incompressible conditions. They found a large difference between the Reynolds and Stokes equations in predicting load when the film thickness to the wavelength ratio is of order of 0.1. Recently, Almqvist and Larsson [10] investigated the flow in a lubricant film on the surface roughness scale and compared the numerical solutions obtained by two different solution approaches, firstly by the CFD (computational fluid dynamic) approach and secondly by the Reynolds equation approach. In the CFD approach, the momentum and continuity equation are used in their basic form, which means that no assumption about neglecting inertia or approximations due to thin lubricating films are used. The results showed that the discrepancies between the two approaches may occur, primarily due to a singularity which appears in the momentum equations when the stresses in the lubricant attain magnitudes that are common in EHL. This singularity is not represented by Reynolds equations. Whereas a singularity might appear in the momentum equation at high shear stresses [11]. It was also shown that in relation to the ratio of the film thickness to the wavelength the Reynolds equation is valid until this ratio is approximately of order of 10^{-2}.

Bair *et al.* [11] pointed out that there is another limitation to the validity of the Reynolds equation, namely, the pressure dependence of the viscosity, and the density of the lubricant. The limitations exist to the validity of the Reynolds equation related to the compressibility of the medium has been also demonstrated by Odyck and Venner [8]. It was shown that the compressibility can still lead to a cross-film pressure dependence which is predicted by the Stokes solution and it is not by the Reynolds solution.

The limitations of the validity of the Reynolds equation in modeling and simulation of the lubrication between real (rough) surfaces have been explored by many workers. However, in deriving the Reynolds equation, the assumption of the no-slip boundary condition was still used.

4. Slip Boundary Condition

In the classical fluid mechanics, it is usually assumed that there is no-slip of the velocity of fluid over a solid wall. This is known as the no-slip boundary condition and has been used in both scientific researches and engineering applications for hundreds of years. It states that fluid adjacent to a solid boundary has zero velocity relative to the solid surface. It also means that the shear stress at the interface between solid and liquid can reach any large value.

For most practical applications, the no-slip boundary condition is a good model for predicting fluid behavior. However, a number of researchers have found some evidences of slip on the interface of a solid and a liquid. Recently, with the advancement of the experimental techniques, such as nano particle image velocimetry (NPIV), atomic force microscope (AFM) and surface force apparatus (SFA), boundary slip has been observed not only for a hydrophobic surface [12, 13] but also for a hydrophilic surface [14, 15]. It has also been found that boundary slip occurs not only in a polymer flow [16], but also in a hydrodynamic [17] and elastohydrodynamic [18] lubrication. In addition, the wall slip is suggested in several theoretical simulations using molecular dynamics [19], and is acknowledged in non-Newtonian fluids [20], in non-aqueous Newtonian fluid at the interface with a hydrophobic surface [12], and in an aqueous Newtonian fluid bounded by relatively hydrophilic solid surface [13]. Therefore, the slip evidence has been generally accepted and for certain cases the no-slip boundary condition is not valid.

4.1. Measurement of wall-slip

In recent years with the progress in micro and nanoscale measurement technology, it is possible for scientists to observe the wall slip in a nanometer. There are three techniques so far for detecting the wall slip: nano particle image velocimetry (NPIV), atomic force microscope (AFM) and surface force apparatus (SFA). The NPIV technique is a direct observation method with a measurement precision depending on the size of the nano particles with a slightly low accuracy. The AFM and SFA are the indirect observation techniques based on the assumption that the slip occurs exactly on the interface of solid and liquid. These methods need a high accuracy boundary slip model to induce the slip velocity. The wall slip is usually described by the slip length model [21-23] at low shear rate or by the critical shear stress model [24-26] at high shear rate. Generally speaking, the degree of boundary of slip is shear dependent.

4.2. Slip length model

The most widely used slip model so far is the slip length model. The slip length model, proposed first by Navier [27], stated that the slip velocity is proportional to the liquid shear rate evaluated at the interface. The slip length model uses a length parameter, called slip length, to predict interfacial slip velocity, which is written as:

$$V_s = b\,\dot{\gamma} \qquad (2)$$

where V_s is the slip velocity, b is the slip length which is a constant for the same interface, and $\dot{\gamma}$ is the local shear rate. The boundary condition is evaluated at the surface. The slip length is the distance behind the interface at which the liquid velocity extrapolates to zero. For a pure shear flow, the slip length b can be interpreted as the fictitious distance below the surface where the no-slip boundary condition would be satisfied (Fig. 2). The slip length is shown to be independent of the type of flow and of the channel width, but it is rather related to the fluid organization near the solid, as governed by the fluid-solid molecular interactions [28]. For polymers, b may reach 10 to 100 μm [12].

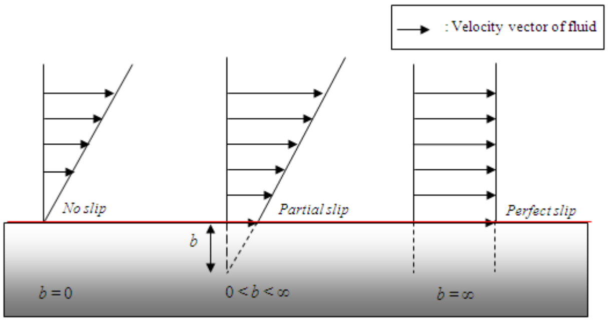

FIGURE 2: *Interpretation of the slip length b, after* [29].

4.3. Critical/shear stress model

In lubrication mechanics, if the lubricant has a critical shear yield stress, for example a viscoplastic fluid, grease, or lubricant at high pressure, wall slip will occurs at the wall/lubricant interfaces when wall shear stress is sufficiently high. The first who considered this critical shear stress in elasto-hydrodynamic lubrication was Smith [30]. Later, it was adopted by Jacobson [31] in a theoretical analysis of a point contact. The analysis contained a non-Newtonian lubricant model considering the solidification of the lubricant, which means that at a certain pressure, referred to as the solidification pressure, the lubricant turns into a glassy state.

Based on the experimental observation and molecular dynamic simulation, Eq. (2) cannot quantitavely describe the interfacial slip velocity. Because, it was found that the parameter b is not a constant at all, especially at high shear rate. At higher shear rates, the Navier condition breaks down as the slip length increases rapidly with $\dot{\gamma}$. The boundary condition is nonlinear even though the liquid is still Newtonian.

The critical shear stress, τ_c is the largest shear stress where the lubricant can sustain, comparable to the yield stress of a metallic material. The critical shear stress depends on the mechanical/chemical properties of the surfaces and the interaction between the surfaces and the lubricant [32]. The critical shear stress, τ_c, is usually given by:

$$\tau_c = \tau_0 + \gamma p \qquad (3)$$

where τ_o is the initial critical shear stress, γ is the critical shear stress proportionally coefficient, and p is the fluid pressure. The parameter γ and τ_0 depend on the chemical composition of the fluid and temperature. $\tau_c = \eta\,\dot{\gamma}$ for a Newtonian fluid, where η is the fluid viscosity. The value of τ_0 used in the literature is usually in the range of 1 to 8 MPa and the value of the proportionally coefficient is the range of 0.03 to 0.15, depending on the type of lubricant. According to the works of Wu and Ma [17], the initial critical shear stress, τ_0, has a larger effect on the wall slip than the critical shear stress proportionally coefficient, γ. In relation to the influence of different lubricants, Stahl & Jacobson [33] also showed that the value of τ_0 has no influence on the film thickness and only a limited influence on the size of the non-Newtonian region.

The shear stress properties of lubricants, especially the critical shear stress, have been measured over the years by using several different methods both static and transient. High pressure chambers, jumping ball apparatus and Kolksky bars are some examples of different techniques that have been used.

The phenomena of wall slip in flowing systems have been addressed in several studies regarding flowing of different polymer melts through conduits and dies. However, some reports have dealt with wall slip in EHL contacts. In Lau *et al.* [16] the wall slip was modelled as a reversible chemical activation process which takes place in an interfacial region between the bulk polymer and the metal surfaces. Brochard *et el*. [34] investigated the wall slip in flowing polymers melts. In their models, polymer chains which are strongly bonded to the contacting surfaces were entangled to the bulk polymer chains. The wall slip was defined as a disentanglement between the chains bonded to the surfaces and the chains in the bulk polymer. This disentanglement appears at a transition point when the wall stress reaches a certain critical value. Hill in [35] presented the wall slip model for polymer melts. The wall slip was defined as the loss of adherence of monomers between the surfaces and the bulk polymers.

In Kaneta *et al*. [18] slip at or near the contacting surfaces was used to explain the occurrence of the strange dimples discovered through the experiments. Furthermore, these strange phenomenons in the results of Kaneta's experiment were described by Ehret [36] by presenting theoretical/numerical analyses with a wall slip model. Their theoretical model is similar to the wall slip model of Stahl & Jacobson [32, 33], including critical shear stress and slip at the interface between the lubricant and the adjacent surfaces.

Choi *et al.* [37] measured pressure drop for flow hydrophobic and hydrophilic microchannels. From discrepancies with the expected pressure drop, slip lengths of tens of nanometers were calculated for both the hydrophilic and hydrophobic surfaces, with the hydrophobic surfaces having slightly larger slip lengths. They investigated experimentally the slip effects of water flow in the hydrophilic/hydrophobic microchannels and found the slip length to vary approximately linearly with the flow shear rate.

Fortier and Salant [38] conducted a numerical analysis of a finite slider bearing with a heterogeneous slip/no-slip surface by means a modified slip length model (a slip length model with a critical shear stress) and found that such a bearing can provide a high load support but low friction drag. However, they meet an instability problem of the numerical solution when the critical shear stress is non-zero and thus conclude that the bearing operates in an unstable condition. In [39]**,** Wu *et al*., based on the critical shear stress model, did not find such a numerical instability. They also showed that with a mixed slip surface, a convergent wedge, a parallel gap, and even a divergent wedge can provide a fluid load support. This breaks through the traditional frame of the classical Reynolds theory, which requires a convergent gap to generate a hydrodynamic pressure. A mixed slip surface is a reasonable combination of the two types of surface, hydrophilic and hydrophobic, which leads to a method to control wall slip. Their numerical studies showed that the hydrodynamic of lubrication film confined between a no-slip surface and a mixed slip surface differs entirely from that of the film confined between two no-slip surfaces.

5.The Degree of Slip

Recent works on slip in the MEMS devices focused on developing materials that will form super-hydrophobic surfaces and also the amount of slip where such surfaces might yields. A variety of computer simulation methods were recently used to understand interfacial boundary conditions in flow of fluids in the MEMS devices.

The degree of slip depends on the factors such as wetting conditions, shear rate, pressure, surface charge, surface roughness, and dissolved gas. In experiments, slip is usually found when the liquid partially wets the solid surface. Measured slip length span four orders of magnitude, from molecular sizes to micrometers, and are usually shear dependent in squeeze flow experiments. Zhu and Granick [21] reported squeeze flow experiments, in which two crossed cylinders oscillate about a fixed average distance. By measuring the viscous resistance, they extracted the slip length over a wide range of oscillation amplitudes. The experiment leads to the largest shear dependent slip length (~ 2 µm). Recently, Joseph and Tabeling [40] measured velocity profiles in water flowing through thin microchannels using particle image velocimetry combined with a nanopositioning system. From the velocity profiles, they determined the slip lengths in two cases: smooth hydrophilic glass surfaces, and smooth hydrophobic glass surface. It was found that the slip length is below 100 nm.

6.Conclusions

The flow dynamics in lubricated-MEMS can be accurately described only if the physics of the flows at the interface with solid surfaces is entirely understood. One important step to reach this purpose consists in determining the correct boundary conditions. The boundary condition for the fluid over a solid surface is the generally accepted no-slip condition. Evidence for slip has recently been collected by different research groups using a range of techniques. Nowadays, a challenging goal for scientists would be to frame the obtained results in an accurate and consistent physical picture of the phenomena occurring at the interface.

The Reynolds equation with no-slip condition is the dominant partial equation used for predicting of the fluid flow when solving elastohydrodynamic lubrication. Very few attempts have been carried out using the modified Reynolds equation with slip. In summary, there are some important points as follows:

1. There are two models to describe the wall slip. First, the slip length model and second, the critical/limiting shear stress model. In fact, the later model has received more attention by recent researchers because its accuracy is better. There is not much attempts to combine for both models to control the wall slip more appropriately.
2. Although it is usually assumed that slip only occurs on hydrophobic surfaces, a large variety of hydrophilic surfaces with different wetting properties have be shown to be prone to slip.
3. The assumption of smooth surface was always used by researchers when proposing a new slip model. It is not always valid at all. This is because at the microscopic level (lubricated-MEMS), the surface is relatively rough and will influence the contact condition. Therefore another approaches need to be employed in order to take the surface roughness into account for presenting a new slip model at the microscale.

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