Stability Analysis of an Adaptive Dominant Type Hybrid Adaptive and Learning Controller for Robot Manipulators

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Abstract

Robot manipulators have been used extensively during several decades in industrial applications in order to increase productivity, flexibility and quality of their product resulted. For the example, we could find it in assembly-line workplace, in which robot manipulators are often employed to follow a motion of desired trajectory that is repeated over a given operation time. Therefore, the robot motion control is one of the key competences for industrial applications of robot manipulators. Then, it is important to increase and improve the motion control method of robot manipulators for achieving an accurate trajectory tracking in order to fulfill all periodic desired trajectory applied in the industrial applications. Further, this paper proposes an adaptive controller dominant type hybrid adaptive and learning controller for trajectory tracking control of the *n* rigid-link robot manipulators. The proposed controller consists of model-based adaptive control (MBAC), repetitive learning control (RLC) and proportional-derivative (PD) control. During the actual position trajectory converges to desired trajectory, the value of adaptive control input increases and becomes dominant by the progress of estimation of dynamical parameters, while the value of the repetitive learning control input decreases close to zero quickly by adding a forgetting factor into learning law. In motion control law, the proposed controller uses only one vector to estimate the unknown dynamical parameters. It makes the proposed controller as a simpler hybrid adaptive and learning controller which does not need much computational power and also is easily be implemented for real applications of robot manipulators. We utilize Lyapunov-direct method based approach to prove stability of the proposed controller because the stability is a fundamental issue in analysis and design of control system, in which the controller can guarantee the position tracking error of robot manipulators converge to zero. Computer simulation results show the effectiveness of the proposed controller in achieving the accurate tracking to the desired trajectory.

Keywords: adaptive control, adaptive dominant, learning control, hybrid control, robot manipulators

Introduction

In industrial applications, we often encounter the automation processes that are done repeatedly in certain time period to produce a product. For instance, the robot manipulators replace human role to perform the same task repeatedly in welding and grinding task. To perform these processes, the tools move quickly, and the control process must be accurate for tracking a prescribed desired trajectory. Therefore, a controller that is capable of improving its accuracy of the repeated task over a given operation time attracts interest of many researchers.

A number of researchers have been developed a hybrid controller which combined the model-based adaptive control (MBAC) with the the repetitive learning control (RLC) as one way to get controllers that had a good accuracy. The basic idea of the MBAC is estimation of the unknown dynamical parameters of manipulator on-line [1]. This controller has proven successful in dealing with estimation of uncertain dynamical parameters during execution of prescribed desired motion accurately. Furthermore, in [2] Arimoto et al. developed the betterment learning controller as the iterative learning controller (ILC). The ILC requested to return to same initial configuration after each learning trial before a new trial can be attempted. Next, a number of research efforts have been made for developing the controller without the requirement for same initial configuration in all learning process that was known as the RLC. To execute the periodic motions in T period, the RLC recalled trajectory error as long executing process and stored this error to update the control input for next period. [3,4] represented this method. Absolutely, by combining the

MBAC and the RLC, the researchers want to obtain their perspective benefits. The RLC will manage control input accurately as long as the desired trajectory that is given as periodic function is not changed, but if it is changed, the controller needs long time to relearn the feed-forward control input. Whereas, applying the MBAC will maintain control input by estimated value of dynamical parameters even if desired trajectory is changed during process occurred.

Many previous papers proposed a hybrid adaptive and learning controller. In [5], the adaptive controller estimated the dynamical parameters of manipulator by adaptation law. In [6], the hybrid controller was proposed with velocity estimation that used a simple linear-state observer to obtain estimates of the joint velocity signal. In [7], Dixon developed a hybrid adaptive/learning controller that utilized learning-based feed-forward terms in based on Lyapunov-based approach. In [8], Nakada proposed a new model of adaptive and learning controller based on [5] in which the controller had more simple condition for stability and resulted the more effective type controller for trajectory tracking. Actually, [5, 8] proposed a learning dominant type hybrid adaptive and learning controller. It means that the RLC input was greater than the MBAC input and the MBAC input was very small when the input torque of the proposed controller archived the actual position trajectory converging to desired trajectory. In [9], Nakada developed the new model hybrid controller in which the MBAC became more dominant than the RLC. The new proposed controller could adjust the feed-forward control input immediately although the desired trajectory was changed. [10] developed the adaptation dominant type hybrid adaptive learning controller based on [9] in which the learning law is added a forgetting factor.

In this paper, we propose a new type of hybrid adaptive and learning controller in which adaptive control input becomes dominant than others when the actual trajectory is tracking close to desired trajectory. The proposed controller has more simple control structure than [10] by using only one estimator of unknown dynamical parameters. We use Lyapunov-direct method to prove stability of the proposed controller, and for ensuring the effectiveness of the proposed controller in achieving the desired trajectory, computer simulations are carried out.

Dynamic Model of Robot Manipulators

The robot manipulator is defined as an open kinematics chain of *n* rigid links. By using the Lagrangian formation, its dynamics model can be described as:

$$R(q)\ddot{q}(t)+\left\{\frac{1}{2}\dot{R}(q)+S(q,\dot{q})\right\}\dot{q}(t)+G(q)=U(t), \quad (1)$$

where q denotes the joint angle position, $R(q)$ represents the inertia matrix, which is symmetric and positive definite, $S(q,\dot{q})$ represents a skewsymmetric matrix from the Coriolis and Centrifugal force, $G(q)$ represents the gravitational force vector, and $U(t)$ represents the control input vector generated by independent torque sources at each joint.

By considering tracking problem, the robot manipulator is required to track the periodic desired trajectory $q_d(t)$ during an interval of finite duration $t \in [0,T]$, where T denotes the period of the desired trajectory. Then, we define the position error as $\Delta q(t) = q(t) - q_d(t)$. Now, we define the filtering tracking error $s(t)\epsilon R^n$ as the difference between current velocity and reference velocity, which is defined as follows:

$$s(t)=\Delta\dot{q}(t)+\alpha\Delta q(t) \quad (2)$$

Based on eq. (1), we can describe another form of equation of motion of robot manipulators that is expressed by:

$$H(q,\dot{q},\dot{q},\ddot{q})\Theta(t)=U(t), \quad (3)$$

where $H(q,\dot{q},\dot{q},\ddot{q})$ is nonlinear function matrix as the regressor matrix that consists of measurable functions of the joint position and joint velocity, while $\Theta(t)$ represents the vector of unknown dynamical parameters such as link masses, moments of inertia and position of mass center of links. Moreover, based on the periodic desired trajectory, we denote the desired regressor matrix as:

$$H_d(q_d,\dot{q}_d,\dot{q}_d,\ddot{q}_d)\Theta(t) \quad (4)$$

And the residual regressor matrix as:

$$H_r(q,\dot{q},\dot{q}_r,\ddot{q}_r)\Theta \quad (5)$$

for

$$\dot{q}_r(t)=\dot{q}_d(t)-\alpha\Delta q(t), \quad (6)$$

and $\dot{q}_r$ is introduced as the nominal reference vector that is used to ensure the convergence of the trajectory tracking, where α is a positive definite matrix. Furthermore, we obtain another form of the dynamic model of the robot manipulator as:

$$R(q)\dot{s}(t)+\left\{\frac{1}{2}\dot{R}(q)+S(q,\dot{q})\right\}s(t)+\bar{H}\Theta+H_d\Theta=U(t) \quad (7)$$

where

$$\bar{H}=H_r(q,\dot{q},\dot{q}_r,\ddot{q}_r)-H_d(q_d,\dot{q}_d,\dot{q}_d,\ddot{q}_d) \quad (8)$$

Adaptive Dominant-Type Hybrid Adaptive and Learning Controller

The objective of this development is to design a controller that can ensure that the position tracking error is asymptotically stable for the periodic desired trajectory. To illustrate the detailed design of the proposed control structure, we consider torque input in the right-hand side of (1) as:

$$U(t) = U_a(t) + U_l(t) + U_{pd}(t) \quad (9)$$

where $U_a(t)$ is the MBAC input, $U_l(t)$ is the RLC input and $U_{pd}(t)$ is the PD control input.

The MBAC input is defined by following equation:

$$U_a(t) = H_r(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{\Theta}(t), \quad (10)$$

where $\hat{\theta}(t) \epsilon R^m$ is a vector that is used to represent a vector of estimated unknown dynamical parameters. The estimation of unknown dynamical parameters are generated on-line according to adaptive update rule by the following adaptation law:

$$\dot{\hat{\Theta}}(t) = -K_a H_r^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)s(t), \quad (11)$$

where K_a is adaptive gain selected as a symmetric positive definite matrix.

The other side, the RLC input recalls the filtering tracking error and stores error to update the next control input. The RLC input is given in the form as the following learning law:

$$U_l(t+T) = \beta U_l(t) - K_l s(t), \quad (12)$$

where β is forgetting factor selected as scalar value $0 < \beta < 1$, whereas K_l is learning gain selected as a symmetric positive definite matrix. The initial learning input is defined by $U_l(0) = 0$, which satisfies $U_l(t)$ is 0 for first period $t \, \epsilon \, [0,T]$. Different from [11], the learning law of the RLC input in eq. (13) is updated only by the filtered error *s(t)* and simpler than the learning law of [10]. Forgetting factor b is used for increasing the convergence speed of input value to zero. The last controller is PD feedback that is defined:

$$U_{pd}(t) = -Ks(t) \quad (13)$$

where K is PD gain selected as a symmetric positive definite matrix.

Stability Analysis

In this section, we provide the proof for asymptotic stability of the proposed controller by using a Lyapunov-like method. Firstly, when we combine the dynamic model of the robot manipulator in eq. (7) with the control input law that is proposed above, we obtain the compact form:

$$\begin{aligned} &R(q)\dot{s}(t) + \left\{\frac{1}{2}\dot{R}(q) + S(q,\dot{q})\right\}s(t) \\ &= (U_a(t) - H_d\Theta) + (U_l(t) - \bar{H}\Theta) - Ks(t). \end{aligned} \quad (14)$$

Then, we define a Lyapunov function candidate *V (t)* $\in R^1$ that is described as the lower bounded function:

$$V(t) = \frac{1}{2}s^T(t)R(q)s(t) + V_a(t) + V_l(t) \geqslant 0, \quad (15)$$

where

$$\begin{aligned} V_a(t) &= \frac{1}{2}\Delta\Theta^T K_a^{-1}\Delta\Theta \\ V_l(t) &= \frac{1}{2}\left\{\int_0^{t+T} U_l^T(\tau)K_l^{-1}\beta^{-1}U_l(\tau)\,d\tau - \int_0^t U_l^T(\tau)K_l^{-1}\beta U_l(\tau)\,d\tau\right\} \\ &= \frac{1}{2}\left\{\int_t^{t+T} U_l^T(\tau)K_l^{-1}\beta^{-1}U_l(\tau)\,d\tau \right. \\ &\quad \left. + \int_0^t U_l^T(\tau)K_l^{-1}\left(\frac{1-\beta^2}{\beta}\right)U_l(\tau)\,d\tau\right\}. \end{aligned} \quad (16)$$

After we substitute eq. (16) into eq. (15), and differentiating $V(t)$ with respect to time, we obtain the following expression:

$$\dot{V}(t) = s^T(t)\left\{R(q)\dot{s}(t) + \frac{1}{2}\dot{R}(q)s(t)\right\} + \dot{V}_a(t) + \dot{V}_l(t). \quad (17)$$

Substituting eq. (14) into eq. (17) and using a fact that matrix $S(q,\dot{q})$ is skew-symmetric, $s^T(t)S(q,\dot{q})s(t) = 0$, we can obtain more simple equation of $\dot{V}(t)$ as follows:

$$\dot{V}(t) = -s^T(t)Ks(t) + \dot{V}_1(t) + \dot{V}_2(t), \quad (18)$$

where

$$\dot{V}_1(t) = (U_a(t) - H_d\Theta)^T s(t) + \dot{V}_a(t) \quad (19)$$

$$\dot{V}_2(t) = (U_l(t) - \bar{H}\Theta)^T s(t) + \dot{V}_l(t). \quad (20)$$

Then, from eq. (8), eq. (10) and eq. (11) through eq. (19), we obtain the simple form of $\dot{V}_1(t)$ that is expressed as:

$$\begin{aligned} \dot{V}_1(t) &= (H_r\hat{\Theta}(t) - H_d\Theta)^T s(t) + \Delta\Theta^T K_a^{-1}\left(-K_a H_r^T s(t)\right) \\ &= (H_r\Delta\Theta + (H_r - H_d)\Theta)^T s(t) - (H_r\Delta\Theta)^T s(t) \\ &= s^T(t)\bar{H}\Theta. \end{aligned} \quad (21)$$

While based on eq. (12), $\dot{V}_l(t)$ in eq. (20) can be expressed:

$$\begin{aligned} \dot{V}_l(t) &= \frac{1}{2}\left\{U_l^T(t+T)K_l^{-1}\beta^{-1}U_l(t+T) - U_l^T(t)K_l^{-1}\beta U_l(t)\right\} \\ &= \frac{1}{2}\left\{(\beta U_l(t) - K_l s(t))^T K_l^{-1}\beta^{-1}(\beta U_l(t) - K_l s(t)) - U_l^T(t)K_l^{-1}\beta U_l(t)\right\} \\ &= \frac{1}{2}\left\{U_l^T(t)K_l^{-1}\beta U_l(t) - U_l^T(t)K_l^{-1}K_l s(t) - s^T(t)K_l^T K_l^{-1}U_l(t) \right. \\ &\quad \left. + s^T(t)K_l^T K_l^{-1}\beta^{-1}K_l s(t) - U_l^T(t)K_l^{-1}\beta U_l(t)\right\} \\ &= -U_l^T(t)s(t) + \frac{1}{2}s^T(t)\beta^{-1}K_l s(t), \end{aligned} \quad (22)$$

and we get value of $\dot{V}_2(t)$ in eq. (20) as follows:

$$\begin{aligned} \dot{V}_2(t) &= (U_l(t) - \bar{H}\Theta)^T s(t) - U_l^T(t)s(t) + \frac{1}{2}s^T(t)\beta^{-1}K_l s(t) \\ &= \frac{1}{2}s^T(t)\beta^{-1}K_l s(t) - s^T(t)\bar{H}\Theta. \end{aligned} \quad (23)$$

Finally, by substituting eq. (21) and eq. (23) into eq. (18), we can express the derivative of Lyapunov function candidate that is another form for rewritten as follows:

$$\begin{aligned} \dot{V}(t) &= -s^T(t)Ks(t) + s^T(t)\bar{H}\Theta + \frac{1}{2}s^T(t)\beta^{-1}K_l s(t) - s^T(t)\bar{H}\Theta \\ &= -s^T(t)\left\{K - \frac{1}{2}\beta^{-1}K_l\right\}s(t). \end{aligned} \quad (24)$$

Based on the above expression, the output error in eq. (24) implies that:

$$\lim_{t\to\infty} s(t) = 0 \quad (25)$$

when we select to satisfy the following sufficient condition:

$$K - \frac{1}{2}\beta^{-1}K_1 > 0, \qquad (26)$$

It is clear that if the controller gains in the proposed controller are selected according to eq. (26), the trajectory tracking error Δq is asymptotically stable.

Computer Simulation

In order to show the effectiveness of the proposed controller, we conducted a computer simulation study that was carried out using a model of a two-link robot manipulator with revolute joints (Fig. 1). The value of physical parameters are set as $m_1 = 0.2\,[kg]$, $l_1 = l_2 = 0.4\,[m]$, $l_{c1} = l_{c2} = 0.2[m]$, whereas the dynamical parameters can be defined as $[\theta]_{1x5}^T = [I_1 + m_1 l_{c1}^2,\ I_2 + m_2 l_{c2}^2,\ m_1 l_{c1}^2,\ m_2 l_{c2}^2,\ m_2]^T$. The gravitational acceleration is 9.81 m/s2 and its direction is toward the negative direction of the Y axis.

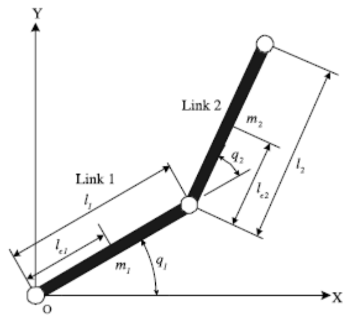

Figure 1. Model of the two-link robot manipulator.

In this simulation, the periodic desired trajectory $qd(t)$ will be performed by the trigonometric function in the joint space which is defined as:

$$q_{d1}(t) = \sin\left(\sin\left(\frac{\pi t}{2}\right)\right) - \frac{1}{2}\sin(\pi t) \quad \text{for } 0 \leqslant t \leqslant 120 \text{ s}$$

$$q_{d2}(t) = \frac{\pi}{3}\left(\sin\left(\frac{2\pi t}{3}\right) - \frac{1}{2}\sin\left(\frac{4\pi t}{3}\right)\right) \quad \text{for } 0 \leqslant t \leqslant 60 \text{ s}$$

$$q_{d3}(t) = \frac{\pi}{\sqrt{27}}\left(2\sin\left(\frac{\pi t}{2}\right) - \sin(\pi t)\right) \quad \text{for } 60 < t \leqslant 120 \text{ s.}$$

The control gains are selected as follows α = diag{3.0, 3.0}, K_a = diag{0.01,0.01, 0.01, 0.01, 0.01}, K_1 = diag{2.9, 2.9}, K = diag{3.0, 3.0} and 0.48 < β <1. We will show simulation results that are the result of using β = 0.5. Furthermore, we simulate the periodic desired trajectory $q_{d1}(t)$ which has period T = 4 s for joint 1.We do not change the desired trajectory over the time interval $t \in [0,\ 120]$ s as the fixed periodic desired trajectory. For joint 2, we employ $q_{d2}(t)$ and $q_{d3}(t)$ that have period T = 3 s and T = 4 s. The $q_{d2}(t)$ traverses time interval $t \in [0,\ 60]$ s and then it is continued by $q_{d3}(t)$ for $t \in [60,\ 120]$. This periodic desired trajectory $q_d(t)$ is shown in Fig. 2.

The simulation of the proposed controller is started by setting zero for all the initial conditions of link angle positions and estimation of unknown dynamical parameters. Fig. 3 shows position tracking error, in which the left-hand side is the error in joint 1 and the right-hand side is the error in joint 2. In joint 1, the position error is reduced each period; in particular, the error shrinks drastically after about 10 s from 0.12 to 0.018 rad. Finally, after 30 periods, the position error closes to zero. The right-hand side of Fig. 3 shows the position tracking error of joint 2 that results from switching over from $q_{d2}(t)$ to $q_{d3}(t)$ at 60 s. When the periodic trajectory tracking is changed, the position error will increase slightly for the next period to 0.003 rad, but it will decrease, return again and converge to zero. It implies that the proposed controller is effective for one or more than one periodic desired trajectory and also the proposed controller guarantees the convergence of the position error to zero.

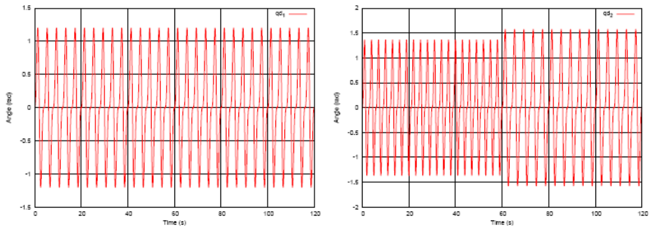

Figure 2. Periodic desired trajectories *qd(t)* of joints 1 and 2.

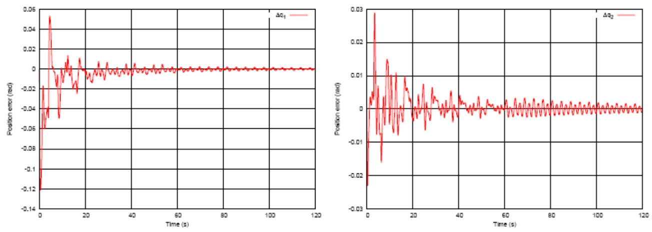

Figure 3. Position tracking error of joints 1 and 2.

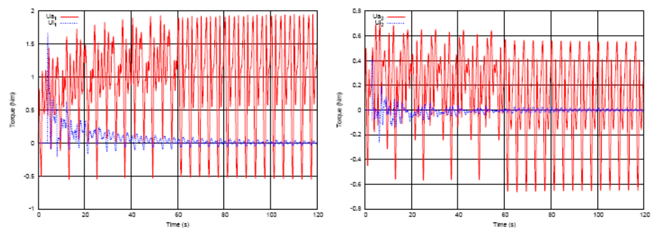

Figure 4. The adaptive and learning control input of joint 1 and joint 2.

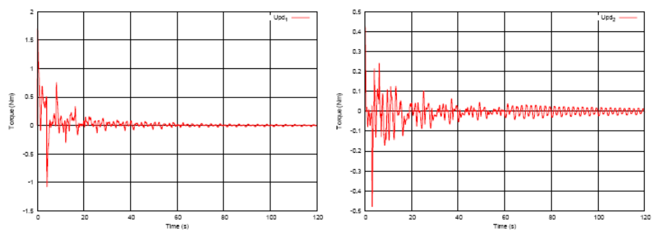

Figure 5. The PD control input of joint 1 and joint 2.

Let us note the adaptive control input shown by the solid line and the learning control input shown by the dotted line of the proposed controller in Fig. 4. The left-hand side shows the adaptive control input and learning control input result in joint 1, in which the initial adaptive control input is small, and then it increases and becomes dominant to achieve the desired trajectory. The highest learning control input is in the second period, and then it decreases according to the learning updated law and it approaches to zero. The right-hand side of Fig. 4 shows the adaptive control input and learning control input, which combine two periodic desired trajectories in joint 2. After switching over the next desired trajectory at 60 s, the adaptive control input adjusts the feed-forward control input immediately by estimating unknown dynamical parameters. Furthermore, it increases again and becomes dominant, while the learning control input does not spend much time to relearn the learning control input, so it not particularly affected by changing the trajectory. When the periodic desired trajectory is changed, the learning law will record the last learning control input for the new period of the desired trajectory. If a new period has a different sampling number of data compared with the old period of the desired trajectory, lacking learning control input for the expanded sampling number will be initialized to zero. However, if the new period has a smaller sampling number, the learning law will only use a necessary number of data according to the requirements of the new period of the desired trajectory. Meanwhile, the PD control input of joints 1 and 2 are shown in Fig. 5. Both PD control inputs are reduced after each period according to the PD control updated law. These conditions show that the PD control input just handles the control input in the early period.

Conclusions

This paper proposes stability analysis of an adaptive dominant-type hybrid adaptive and learning controller for tracking control of a n rigid-link robot manipulator by using Lyapunov-direct method. The proposed controller has been successfully developed to control the joint position of the robot manipulator to achieve high-accuracy trajectory tracking performance. We develop the adaptive dominant-type controller by using only one vector for estimating unknown dynamical parameters, in which it has a simpler control structure than the previous control strategies. The proposed controller is very useful when it is used to track the fixed periodic desired trajectory or the different periodic desired trajectory.

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