

Override and Anti-Windup Control Strategy for Active Vehicle Suspension System

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Abstract

Vehicle suspension plays important roles in vehicle performance. The main function of the vehicle suspension is to improve ride and handling performance. The other important requirements need to be considered in the controller design are suspension deflection and actuator saturation. However, these requirements are conflicting. For example, to obtain better ride comfort it is usually required larger control input and larger suspension deflection, but the actuator that deliver the control signal have a limitation which is commonly known as actuator saturation. There is also a structural constraint that limits the suspension deflection. Most of the vehicle active suspension control strategies deal with actuator saturation and limitation of suspension deflection by keeping the control signals small, until the point where constraint is not met at all. The advantage of this approach is allowing one to use the unconstrained design methods and hence a linear analysis of the problem can be carried out. However, this is achieved at the cost of reducing achievable performance since we expect high performance to be associated with acting on or near constraints. In this study, an alternative approach to the vehicle active suspension system is studied. In this approach, some separation in the controller such that one part is devoted to achieve nominal performance and the other part is devoted to constraint handling is performed. This control strategy involves a two-step design procedure. Namely, a state feedback controller is first synthesized for a linear system ignoring saturation and state constraint. Then, the anti-windup and override compensator is designed such that, when saturation occurs or state constraints limits is exceeded, this compensator becomes active to recovers as much as possible the performance lost. Local control design technique based on the circle criterion and \mathcal{L}_2 gain performance is used for the anti-windup and override compensator synthesis. A quarter car model is considered in this study and the effectiveness of the proposed approach is shown by a numerical example. The application of the override control for active vehicle suspension system is studied. It is shown that the occurrence of the suspension hits its deflection limits and control input reaches the saturation bound can be minimized by override and anti windup control strategy.

Key Words: Feedback Control, Override Control, Anti-Windup Control, Active Vehicle Suspension, Saturation.

Introduction

More and more systems are involved in today cars to guarantee both safety (e.g. ABS, ESP, etc) and comfort (e.g. suspension control, cabin noise control, etc). Concerning the vehicle comfort and road holding ability, the suspension system has the major roles to provide 1) isolating passengers from vibration and shock arising from road roughness (ride comfort); 2) suppressing the hop of the wheels so as to maintain firm and uninterrupted contact of wheels to road (good handling or good road holding); and 3) keeping suspension strokes within an allowable maximum (Chen and Guo, 2005). The most important objective for the vehicle suspension system is the improvement of the ride comfort (Sun et al., 2011) and a significant control input is often necessary to obtain better performance. However, in practice, the actuators which deliver the control signal are always subject to limits in their magnitude which is commonly known as actuator saturation. In addition, there is also a structural constraint that limit the suspension deflection.

There are several ways of dealing with the control and output constraints. One can use the control and output constraints as optimization constraints such as used

in (Chen and Guo, 2005; Sun et al., 2011). This leads to a quite significant linear programming problem and a controller can be designed such that it ensures that the state always belongs to the maximum output admissible set. The finite frequency \mathcal{H}_∞ control has been proposed in (Sun et al., 2011) which shows that an improvement in ride comfort is obtained compared with the entire frequency \mathcal{H}_∞ control. Although the aforementioned controllers have been designed so as to maintain the suspension deflection in a certain range based on a regulated road disturbance, the control input and the suspension deflection might reach the limitation due to various road shapes. One way of incorporating output constraints into controller design is using model predictive control (MPC) strategies. However, the MPC approach is generally expensive in terms of computation (Turner and Postlethwaite, 2002). The other way of dealing with the output constraints is override control (Turner and Postlethwaite, 2002, 2004; Park and Youn, 2003), and in this study, we consider the override control to tackle the problem of the output constraints.

In (Wasiwitono and Saeki, 2011) an alternative approach for the vehicle active suspension system is proposed. A separation in the controller such that one part is devoted to achieve nominal performance and the

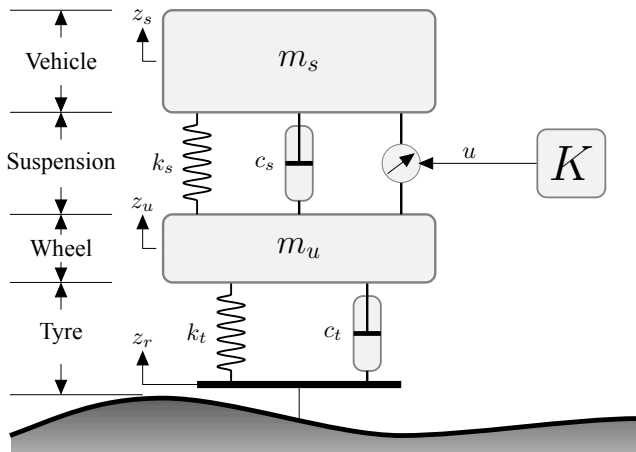


Fig. 1: Quarter car model

other part is devoted to constraint handling is performed. This is the approach taken in anti-windup compensation (Tarbouriech and Turner, 2009). It is shown that the proposed control approach has the potential benefit in achieving the best possible ride comfort. Based on the two-step design procedure, in this study, the combination of override control and anti-windup control strategy are considered to deal with output constraint and control input constraint. Local control design technique based on the circle criterion and \mathcal{L}_2 gain performance is used for the anti-windup and override compensator synthesis.

Further, the paper is organized as follows. Section II describes the quarter car system and problem formulation. Section III describes the override and anti-windup compensator synthesis and a numerical example is given in Section IV to show the usefulness of the proposed control strategy.

Problem Formulation

The quarter car model shown in Fig. 1 is considered in this study. The quarter-car model is very often used for the vehicle suspension analysis and design, because of its simplicity yet capture many important characteristics of the full model. In Fig. 1, m_s is the sprung mass, which represents the car chassis; m_u is the unsprung mass, which represents mass of wheel assembly; c_s and k_s are damping and stiffness of the suspension system, respectively; k_t and c_t stand for compressibility and damping of the pneumatic tire, respectively; z_s and z_u are the displacements of the sprung and unsprung masses, respectively; z_r is the road displacement input; and u is the active input of the suspension system.

Define the following state variables:

$$x_1 = z_s - z_u, \quad x_2 = z_u - z_r, \quad x_3 = \dot{z}_s, \quad x_4 = \dot{z}_u \quad (1)$$

where x_1 denotes the suspension deflection, x_2 denotes the tire deflection, x_3 denotes the sprung mass velocity, and x_4 denotes the unsprung mass velocity. Further, define the disturbance input as $d = \dot{z}_r$, and $x_p = [x_1 \ x_2 \ x_3 \ x_4]^T$, then, by applying Newton's second law of motion and using the static equilibrium position as the origin, the state-space form of the vehicle suspension system can be written as

$$\dot{x}_p = A_p x_p + B_d d + B_u u \quad (2)$$

where $x_p \in \mathcal{R}^{n_p}$ is the suspension system state, $d \in \mathcal{R}^{n_d}$ is the disturbance input, $u \in \mathcal{R}^{n_u}$ is the control input, and

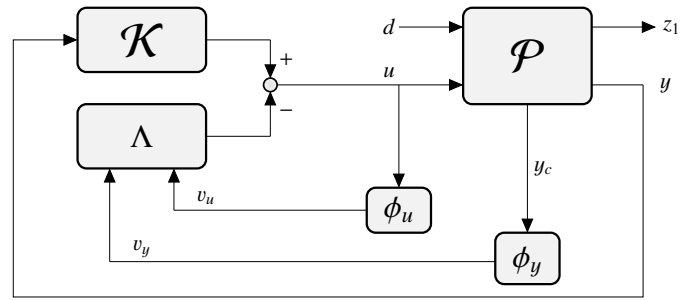


Fig. 2: State feedback control system with deadzone loop and override control strategy

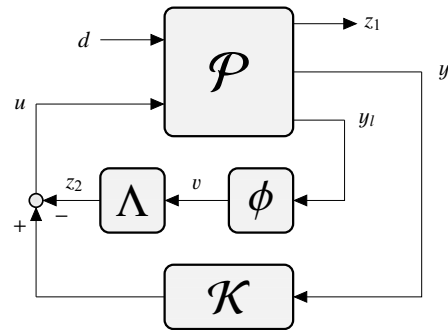


Fig. 3: Override control strategy

$$A_p = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \\ \frac{k_s}{m_u} & -\frac{k_t}{m_u} & \frac{c_s}{m_u} & -\frac{c_s+c_t}{m_u} \end{bmatrix} \quad (3)$$

$$B_d = \begin{bmatrix} 0 \\ -1 \\ 0 \\ \frac{c_t}{m_u} \end{bmatrix}, \quad B_u = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_s} \\ -\frac{1}{m_u} \end{bmatrix}$$

In this study the application of the override and anti-windup control strategy for an active vehicle suspension as shown in Fig. 2 is studied. In this Figure y_c is the constrained output, ϕ_u and ϕ_y is deadzone function to control the activation of the compensator Λ . Further, by defining

$$C_{yl} = \begin{bmatrix} C_{y_c} \\ 0 \end{bmatrix}, \quad D_{yu} = \begin{bmatrix} 0 \\ I_{n_u} \end{bmatrix} \quad (4)$$

the closed-loop system shown in Fig. 2 can be represented as that shown in Fig. 3, with the plant \mathcal{P} is described as

$$\dot{x}_p = A_p x_p + B_u u + B_d d \quad (5)$$

$$z_1 = C_{z1} x_p + D_{zu1} u + D_{zd1} d \quad (6)$$

$$y_l = C_{yl} x_p + D_{yu} u \quad (7)$$

Assume that the linear control \mathcal{K} has been designed, let now focused on the controlled output for the design of the override and anti-windup compensator Λ . The most important objective of the active vehicle suspension is the improvement of the ride comfort, hence, the sprung mass acceleration \ddot{z}_s is chosen as performance output and we have

$$C_{z1} = \begin{bmatrix} -\frac{k_s}{m_s} & 0 & -\frac{c_s}{m_s} & \frac{c_s}{m_s} \end{bmatrix} \quad (8)$$

$$D_{zu1} = \begin{bmatrix} \frac{1}{m_s} \end{bmatrix}, \quad D_{zd1} = \begin{bmatrix} 0 \end{bmatrix} \quad (9)$$

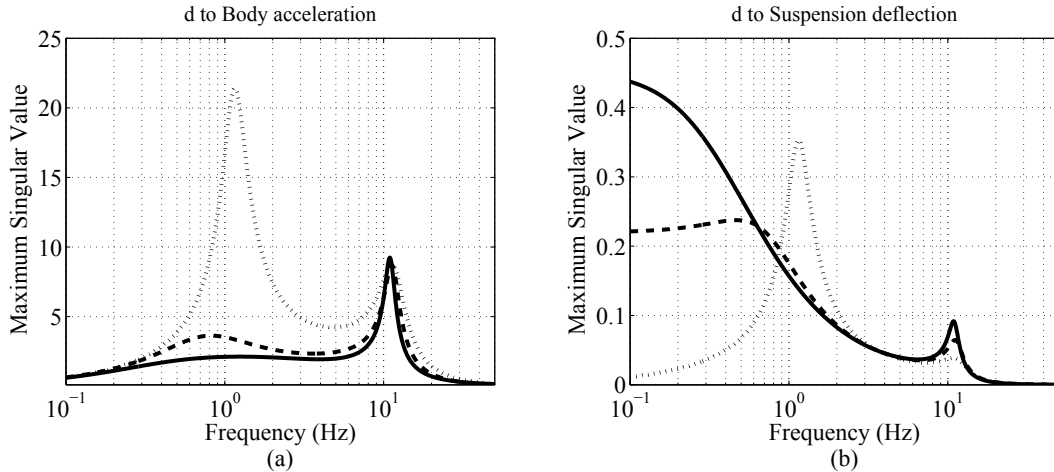


Fig. 5: Frequency response from disturbance to body acceleration and suspension deflection (dotted line: passive suspension; dashed line: state feedback control (Chen and Guo, 2005); solid line: state feedback control (Sun et al., 2011))

where A_m represents the amplitude of the bump and f is disturbance frequency. Fig. 6 shows the plots of body acceleration, suspension deflection and control input for the case of disturbance frequency 1 Hz and bump amplitude 0.1 m. It can be seen that the ride comfort by state feedback control (Sun et al., 2011) is better compared with that by state feedback control (Chen and Guo, 2005). However, the improvement in the ride comfort requires a larger actuator force and larger suspension deflection. Fig. 7 shows the plots of body acceleration, suspension deflection and control input for the case of disturbance frequency 1 Hz and bump amplitude 0.16 m. It can be seen in this Figure that the ride comfort by state feedback control (Sun et al., 2011) is deteriorated caused by the suspension hits its limits, at around 0.5 s.

To overcome this problem, we add override and anti-windup compensator and based on Theorem 1, we synthesize the compensator Λ by setting $\kappa = 1$, $W_1 = 250$ and $W_2 = 50$, and obtain the following result

$$\Lambda = [2.55657 \quad -18.72049] \quad (22)$$

Since the control input saturation is ± 1.5 kN and suspension deflection limit is ± 0.1 m, we choose the dead-zone limit $\phi_u = \pm 1$ kN and $\phi_y = \pm 0.05$ m, respectively. Fig. 8 shows the plots of body acceleration, suspension deflection and control input for the case of disturbance frequency 1 Hz and bump amplitude 0.16 m with proposed control strategy. It can be seen that the improvement in ride comfort is obtained. The suspension deflection and control input is reduced to be below the allowable bound.

Further, to compare effectiveness of the proposed control strategy with standard state feedback control (Chen and Guo, 2005; Sun et al., 2011) at different disturbance frequencies, Fig. 9 shows the plots of body acceleration, suspension deflection and control input for the case of disturbance frequency 4 Hz with bump amplitude 0.11 m. It can be seen from these Figure that both the state feedback control (Chen and Guo, 2005; Sun et al., 2011) hit the suspension deflection limit at around 0.1 s. Whilst, the active suspension system with proposed control strategy does not.

Conclusion

The application of the override and anti-windup control for active vehicle suspension system is studied. It is shown that the occurrence of the suspension hits its deflection limits and control input reaches the saturation bound can be minimized by the proposed control strategy. Furthermore, by setting $M = KQ$ in (19) it is possible to synthesize the state feedback controller K and compensator Λ simultaneously. It is our future work to show the usefulness of such simultaneous design algorithm.

A Proof of Theorem 1

Consider a quadratic Lyapunov function

$$V(x_p) = x_p^T P x_p, \quad P = P^T > 0, \quad P \in \mathbb{R}^{n_p \times n_p} \quad (23)$$

In order to show that the closed loop system is asymptotically stable and the \mathcal{L}_2 gain from d to z is less than γ , we may show that the Lyapunov function (23) satisfies the next dissipation inequality

$$\frac{dV}{dt} < \gamma d^T d - \frac{1}{\gamma} z^T z \quad (24)$$

By using the sector condition (18) and the S-procedure, we obtain

$$\frac{dV}{dt} + \frac{1}{\gamma} z^T z - \gamma d^T d - 2v^T X(v - \kappa y_l) < 0 \quad (25)$$

Define $\xi = [x_p^T \quad v^T \quad d^T]^T$, (25) can be written in the form

$$\xi^T \left[\begin{array}{c} (A_p^T + K^T B_u^T)P + P(A_p + B_u K) + \frac{1}{\gamma}(C_z^T + K^T D_{zu}^T)(C_z + D_{zu}K) \\ -\Lambda^T B_u^T P + X\kappa(C_{yl} + D_{yu}K) + \frac{1}{\gamma}\Lambda^T D_{za}^T(C_z + D_{zu}K) \\ B_d^T P + \frac{1}{\gamma}D_{zd}^T(C_z + D_{zu}K) \\ -PB_u\Lambda + (C_{yl}^T + K^T D_{yu}^T)\kappa X + \frac{1}{\gamma}(C_z^T + K^T D_{zu}^T)D_{za}\Lambda \\ -2X - \Lambda^T D_{yu}^T \kappa X - X\kappa D_{yu}\Lambda + \frac{1}{\gamma}\Lambda^T D_{za}^T D_{za}\Lambda \\ \frac{1}{\gamma}D_{zd}^T D_{za}\Lambda \\ PB_d + \frac{1}{\gamma}(C_z^T + K^T D_{zu}^T)D_{zd} \\ \frac{1}{\gamma}\Lambda^T D_{za}^T D_{zd} \\ -\gamma I + \frac{1}{\gamma}D_{zd}^T D_{zd} \end{array} \right] \xi < 0 \quad (26)$$

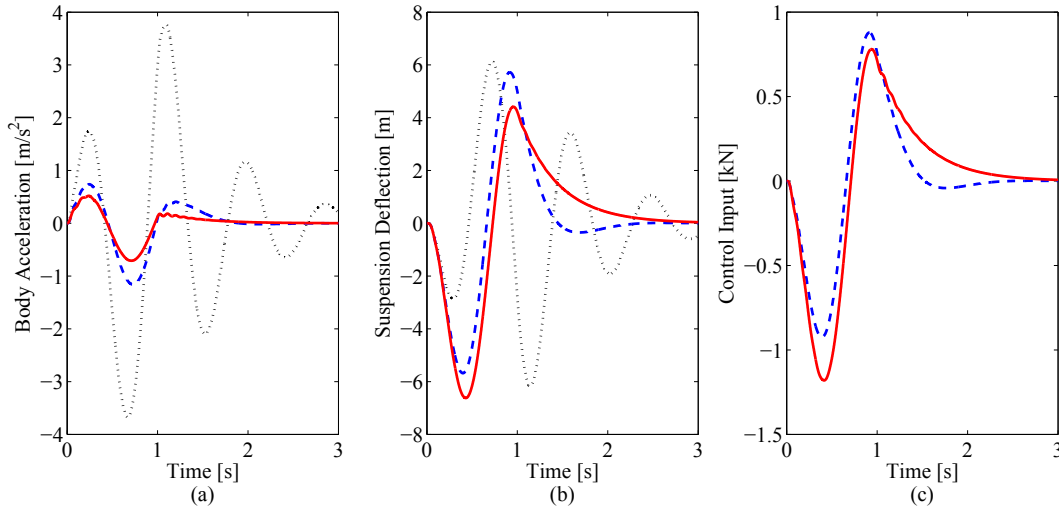


Fig. 6: Bump response for the case of disturbance frequency 1 Hz and amplitude 0.1 m (dotted line: passive suspension; dashed line: state feedback control (Chen and Guo, 2005); solid line: state feedback control (Sun et al., 2011))

Since (26) must hold for all x_p, v , and d , the matrix $P > 0$ must satisfy the next constraint

$$\begin{bmatrix} (A_p^T + K^T B_u^T)P + P(A_p + B_u K) + \frac{1}{\gamma}(C_z^T + K^T D_{zu}^T)(C_z + D_{zu}K) \\ -\Lambda^T B_u^T P + X\kappa(C_{yl} + D_{yu}K) + \frac{1}{\gamma}\Lambda^T D_{za}^T(C_z + D_{zu}K) \\ B_d^T P + \frac{1}{\gamma}D_{zd}^T(C_z + D_{zu}K) \\ -PB_u\Lambda + (C_{yl}^T + K^T D_{yu}^T)\kappa X + \frac{1}{\gamma}(C_z^T + K^T D_{zu}^T)D_{za}\Lambda \\ -2X - \Lambda^T D_{yu}^T \kappa X - X\kappa D_{yu}\Lambda + \frac{1}{\gamma}\Lambda^T D_{za}^T D_{za}\Lambda \\ \frac{1}{\gamma}D_{zd}^T D_{za}\Lambda \\ PB_d + \frac{1}{\gamma}(C_z^T + K^T D_{zu}^T)D_{zd} \\ \frac{1}{\gamma}\Lambda^T D_{za}^T D_{zd} \\ -\gamma I + \frac{1}{\gamma}D_{zd}^T D_{zd} \end{bmatrix} < 0 \quad (27)$$

Applying the Schur complement to (27), we obtain

$$\begin{bmatrix} A_p^T P + PA_p + K^T B_u^T P + PB_u K \\ -\Lambda^T B_u^T P + X\kappa C_{yl} + X\kappa D_{yu} K \\ B_d^T P \\ C_z + D_{zu} K \\ -PB_u\Lambda + C_{yl}^T \kappa X + K^T D_{yu}^T \kappa X & PB_d & C_z^T + K^T D_{zu}^T \\ -2X - \Lambda^T D_{yu}^T \kappa X - X\kappa D_{yu}\Lambda & 0 & \Lambda^T D_{za}^T \\ 0 & -\gamma I & D_{zd}^T \\ D_{za}\Lambda & D_{zd} & -\gamma I \end{bmatrix} < 0 \quad (28)$$

Applying a simple congruence transformation block-diag(P^{-1}, X^{-1}, I, I) to (28) and define $Q = P^{-1}$, $T = X^{-1}$, we have

$$\begin{bmatrix} QA_p^T + A_p Q + QK^T B_u^T + B_u K Q \\ -T\Lambda^T B_u^T + \kappa C_{yl} Q + \kappa D_{yu} K Q \\ B_d^T \\ C_z Q + D_{zu} K Q \\ -B_u \Lambda T + QC_{yl}^T \kappa + QK^T D_{yu}^T \kappa & B_d & QC_z^T + QK^T D_{zu}^T \\ -2T - T\Lambda^T D_{yu}^T \kappa - \kappa D_{yu}\Lambda T & 0 & T\Lambda^T D_{za}^T \\ 0 & -\gamma I & D_{zd}^T \\ D_{za}\Lambda T & D_{zd} & -\gamma I \end{bmatrix} < 0 \quad (29)$$

Define $N = \Lambda T$ then (29) can be written as (19)

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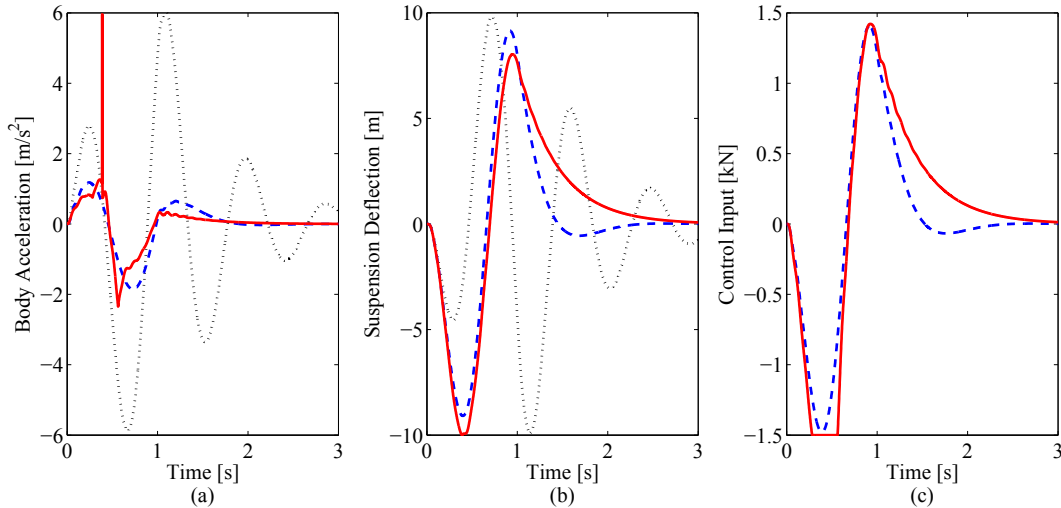


Fig. 7: Bump response for the case of disturbance frequency 1 Hz and amplitude 0.16 m (dotted line: passive suspension; dashed line: state feedback control (Chen and Guo, 2005); solid line: state feedback control (Sun et al., 2011))

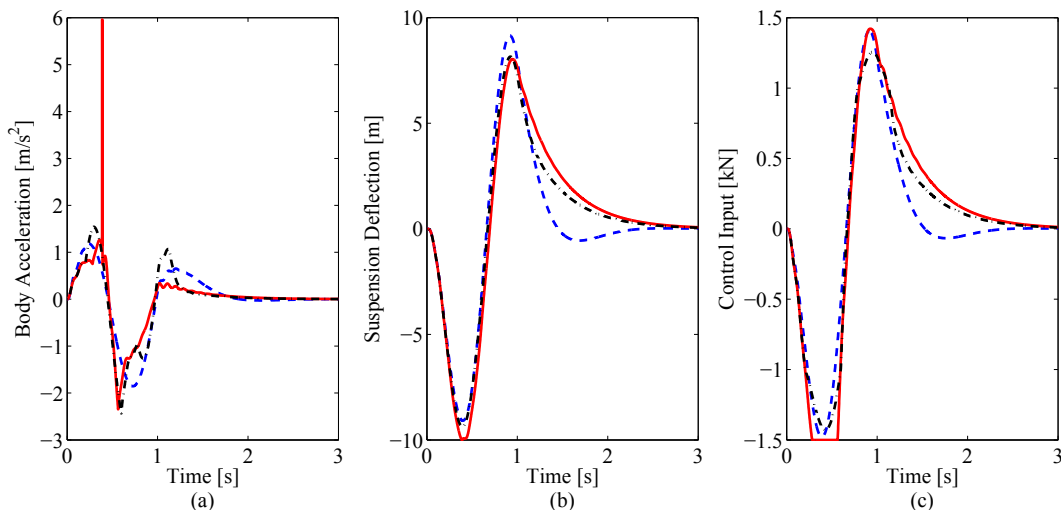


Fig. 8: Bump response for the case of disturbance frequency 1 Hz and amplitude 0.16 m (dashed line: state feedback control (Chen and Guo, 2005); solid line: state feedback control (Sun et al., 2011)); dash-dotted line: proposed control strategy)

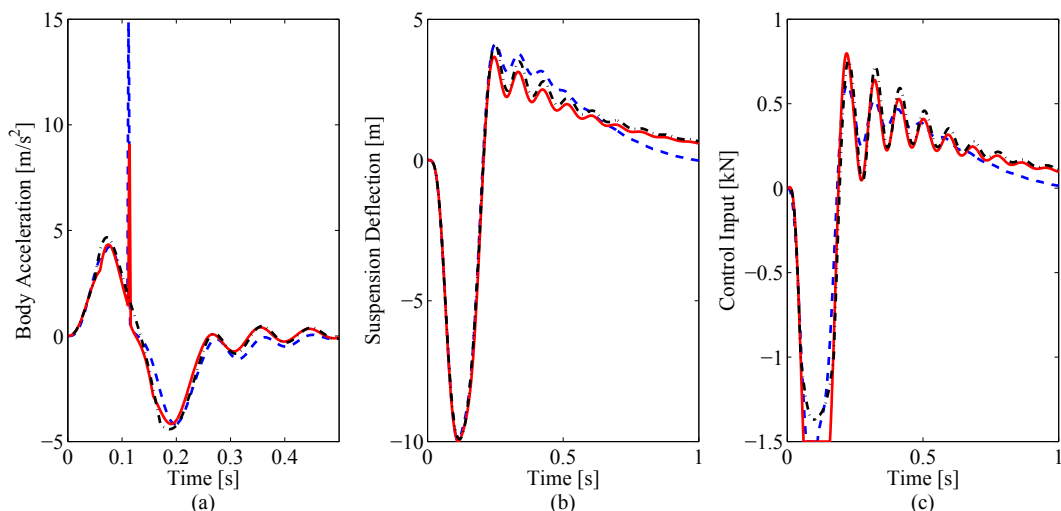


Fig. 9: Bump response for the case of disturbance frequency 4 Hz and amplitude 0.11 m (dashed line: state feedback control (Chen and Guo, 2005); solid line: state feedback control (Sun et al., 2011)); dash-dotted line: proposed control strategy)