Evaluation of sub-pixel accuracy characteristics on Digital Image Correlation
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Abstract
Digital Image Correlation (DIC) is one of the optical methods for non-intrusive experiment in solid mechanics. In order to achieve a better accuracy, sub-pixel interpolation should be implemented in this method. In this paper, the characteristics of the sub-pixel accuracy are evaluated by using Gaussian interpolation method. The characteristics evaluations were performed for DIC with both artificial (simulation) and experimental image. By assuming the speckle on image is a circle, the size parameter of the speckle was represented by its radius. Variation of the final speckle radius on simulation was randomly determined by the seeding of the individual speckle radius and position on the generated artificial images. From the simulation results, the sub-pixel accuracy was constant in every integer displacement. However, in term of the relative error, the percentage value would be lower in higher displacement. In this point, the minimum integer displacement of the specimen on the image can be determined in order to have a certain level of the error. This simulation result has an important practical recommendation to perform DIC experiment, e.g. in order to achieve 3% of the relative error, the load to the specimen should be able to give minimum displacement of 8(eight) pixel on the image. Experimental translation tests were also carried out in order to verify this practical recommendation.

Keywords: DIC, experiment, accuracy, sub-pixel, displacement.

Introduction
Digital Image Correlation (DIC) has been widely used as the optical methods for non-intrusive measurement in solid mechanics [1][2]. In 2D-DIC, there are a camera, a speckled-specimen and processing unit (computer). By using recorded images of undeformed and deformed specimen, full-field displacement can be obtained. Displacement field can be used to determine stress-strain field. In fracture mechanics, this method has been used to analyze stress intensity factor on a specimen [3][4][5].

Beside the growing practical application, accuracy of the method also becomes the main concern of the DIC’s development. Accuracy evaluations of the DIC such as optimal subset size [6] and comparison between Normalized Cross-Correlation (NCC) with Fast Fourier Transformation (FFT)-based correlation [7] have been performed. However, practical questions about the best spraying technique to create the speckled-specimen and the best distance between camera and specimen are still very difficult to answer due to dependency to the test-case.

This paper attempts to present the practical recommendation to eliminate those difficult practical questions. In order to do that, we need to find a relationship between the basic of digital image and structural analysis properties, which are pixel and displacement. Two characteristics which are effected the DIC’s accuracy in pixel are speckle radius and subset-template ratio.

Evaluation Method
By using DIC with artificial image and then verified the result with experiment, the evaluation of the sub-pixel accuracy characteristics was performed in this work.

1. DIC with artificial image

DIC as the experimental technique was simulated in the computer. Thus, recorded images were generated artificially. These images were correlated to find the integer displacement. The accuracy of this displacement was increased by the sub-pixel interpolation. Evaluation was then performed by using error analysis.

a. Artificial image

Artificial images in the present work were generated by using Gaussian function such as

\[ I_p(x,y) = I_i \exp \left( -\frac{g}{4\sigma^2} ((x-x_i)^2 + (y-y_i)^2) \right) \] (1)
The known parameters on equation (1) were subscripted by $i$ such as maximum intensity on the center position ($I_i$), diameter ($d_i$) and center position ($x_i, y_i$). These parameters described the shape and intensity distribution of a single dot on generated artificial image. Intensity distribution generated a grayscale image.

Figure 1 shows the scaled image of the generated dots with individual diameter equal to 10 pixel and the maximum intensity on the center is 255. Image size is 80×80 pixel². There are six center positions of dots that are generated on the image. These positions were randomly distributed. Hence, the overlapping dots were occurred.

Speckle is the cumulative form of the connected dots. It can be identified by converting grayscale image into binary image. In the present work, the speckle was assumed to be circular and its radius represented the length of it. Any overlapping dots were count as one speckle. In the case of Figure 1, instead of six speckles, there are four speckles on the image due to the overlapping condition.

Artificial images were generated for undeformed and deformed condition. Deformed condition represented the uniform translation of the dots. Image size of the present work was 256×256 pixel². Inside this image, there were 3000 dots. Individual speckle radius was varied from 1 to 10 pixel in six range of simulations, marked by the range of dot-radius ($rd$). In case of $rd$ equal to 1-5 pixel, there were dots on image with random radius from 1 to 5 pixel, as can be seen in scaled image in Figure 2.

b. Correlation

Normalized cross-correlation method was used in order to find integer displacement from the artificial images. Small subset ($A$) on the undeformed image and larger template ($B$) on the deformed image were correlated to find the highest correlation value and position.

The normalized cross-correlation value ($C$) can be calculated by using such as

$$C(i,j) = \frac{\sum_{m,n} A(m,n)B(m+i,n+j)}{\sqrt{\sum_{m,n} A^2(m,n)\sum_{m,n} B^2(m+i,n+j)}}$$  \hspace{1cm} (2)

In the present work, the template size was 100 pixel² while the subset sizes were varied from 10 to 80 pixel². Subset-template ratio ($Rst$) was made to be dimensionless to these variations. This ratio represented the sampling area of the speckle in the correlation process.

c. Sub-pixel interpolation

Generated artificial image has a Gaussian intensity distribution. Therefore, Gaussian interpolation was used as the sub-pixel interpolation method. For two-dimensional case, the interpolation can be expressed as

$$dx' = dx + \frac{\ln C(i-1,j)-\ln C(i+1,j)}{2(\ln C(i+1,j)-2\ln C(i,j)+\ln C(i-1,j))}$$  \hspace{1cm} (3)

$$dy' = dy + \frac{\ln C(i,j-1)-\ln C(i,j+1)}{2(\ln C(i,j+1)-2\ln C(i,j)+\ln C(i,j-1))}$$  \hspace{1cm} (4)

where $dx$ and $dy$ are the integer displacement in x and y axis respectively and $C$ is the matrix of the correlation value.

d. Error analysis

The main evaluation in this paper for the sub-pixel accuracy is the mean absolute error (MAE) and relative error (RE). It can be written as

$$MAE = \varepsilon = |d_s - d_r|$$  \hspace{1cm} (5)

$$RE = \frac{\varepsilon}{d_r}$$  \hspace{1cm} (6)
Mean absolute error in equation (5) represented the deviation of the measured simulation data ($d_s$) to the known displacement ($d_r$). In the present work, this error is presented in pixel. Meanwhile, relative error in equation (6) is defined as a comparison between the mean absolute error to the known displacement ($d_r$). This error, in percent, is represented the accuracy of the measurement.

2. DIC with experimental image

The experiment was performed in order to verify the result of sub-pixel accuracy evaluation based on artificial image. It was conducted by placing a camera perpendicular to a calibration board or a specimen.

In Figure 3(left), a printed-paper with chessboard pattern was used for calibration in order to get the conversion factor in [mm/pixel], while in Figure 3(right) a black-painted acrylic specimen was white-sprayed in order to create speckled-surface.

Experimental setup consists of a camera, which was arranged perpendicular to the experimental object (calibration board or specimen). The object was connected to the micrometer slider by a holder. The schematic figure of this experimental setup can be seen in Figure 4.

In the experiment, the specimen was translated from 0(zero) mm to 0.9 mm with 0.1 mm increment in x-direction by using micrometer. Images were captured on each translation increment. To find the sub-pixel displacement, the image from each translation was then paired with image from zero translation.

Experimental image has the resolution up to 8 Megapixel. However, in order to have the same image size as the artificial image on simulation, the experimental images were cropped into 256×256 pixel$^2$. The same algorithms of correlation and sub-pixel interpolation were implemented to these experimental images.

**Figure 3.** Experimental object: calibration board (left), speckled-specimen (right).

**Figure 4.** Schematic of the experimental setup.
Results and discussion

There are two important parameters on the speckle, which are its number and radius. These parameters differ for each subset-template ratio ($R_{st}$). Average distribution of those parameters can be seen in Figure 5. In Figure 5(a), the average speckle radius for bigger subset-template ratio is increasing with the range of the dot radius ($rd$). In smaller $R_{st}$, the trend was almost constant due to the smaller sampling area of the subset. For middle range of the ratio, e.g. 0.4 to 0.6, the average speckle radius is almost linearly increasing, following the linear increment of the average of $rd$.

In Figure 5(b), the average speckle number shows an opposite trend with the speckle radius. Increasing $rd$ decreases the number of speckle on image. On each range, the bigger subset-template ratio, the higher average speckle number. In smaller $R_{st}$, e.g. 0.1, due to small sampling area, the number of speckle was almost constant for all range of dot-radius. Figure 5 shows that for all $R_{st}$, $rd$ equal to 1-5 pixel has the average speckle radius tend to approach the same value and clear different value of average speckle number. Hence, effect of $R_{st}$ to the displacement can be analyzed in this range of dot-radius.

![Figure 5. Speckle parameters: (a) average radius, (b) average number.](image-url)
Figures 6 shows the effect of different $R_{st}$ to the sub-pixel displacement in range of dot-radius equal to 1-5 pixel. In Figure 6(a), mean absolute error of the displacement is lower in bigger $R_{st}$, thus higher accuracy. This condition probably influenced by the higher number of speckle, which increase the sampling speckle for the correlation. This sub-pixel accuracy trend is constant in every integer displacement. As the consequence, the higher displacement value, the percentage of the relative error is decrease, thus produces higher accuracy, as shown in Figure 6(b).

As shown in Figure 6(a), $R_{st}$ equal to 0.1 has the highest mean absolute error. This finding can be used in order to get the minimum integer displacement for a certain percentage of relative error. Bigger $R_{st}$ would have a lower minimum displacement. Thus, result from $R_{st}$ equal to 0.1 can be used as the upper border of the minimum integer displacement.

![Figure 6](image_url)
Effect of range of dot-radius ($rd$) to the sub-pixel displacement is shown in Figure 7. In the Figure 7(a), smaller dot-radius range ($rd$) produces lower error, thus improves the accuracy. It is probably influenced by the higher number of speckle, due to smaller dot size, which increases the sampling speckle for the correlation. Due to its constant trend in integer displacement, the percentage of the relative error can be plotted as in the Figure 7(b).

Figure 7(b) represents the possible overall accuracy of the simulation, which can be used in practical implementation as an upper border of the accuracy of the sub-pixel displacement. In that figure, it can be observed that for all range of $rd$, to obtain better accuracy than 3% relative error, displacement on images should be more than 8 pixel.

![Figure 7](image-url)

**Figure 7.** Effect of range of dot-radius ($rd$) to the sub-pixel displacement ($dx$) on subset-template ratio ($Rst$) equal to 0.1: (a) mean absolute error, (b) relative error.
2. DIC with experimental image

Experimental images were recorded and analyzed in order to verify the result and recommendation of DIC with artificial image (simulation). Several properties of the experimental image can be seen in Table 1.

Table 1. Experimental image properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Image size</td>
<td>256x256 pixel²</td>
</tr>
<tr>
<td>Subset-template ratio</td>
<td>0.1</td>
</tr>
<tr>
<td>Template size</td>
<td>100x100 pixel²</td>
</tr>
<tr>
<td>Conversion factor</td>
<td>0.144 mm/pixel</td>
</tr>
<tr>
<td>Average speckle radius</td>
<td>2.62 pixel</td>
</tr>
<tr>
<td>Average speckle number</td>
<td>2.69</td>
</tr>
</tbody>
</table>

In order to compare with the simulation result, known displacement in millimeter was converted into pixel by using conversion factor from calibration. Average speckle radius on the experimental image was in the range of rd equal to 1-5 pixel. This comparison used Rs equal to 0.1 and error analysis of the simulated DIC can be referred to Figure 6.

Figure 8 presents the comparison of the DIC with artificial and experimental image. In that figure, the relative error of the experimental data decreases in higher displacement. Comparison data of relative error from DIC with artificial image was taken from the nearest sub-pixel displacement data. Decrement trend of the simulation, as shown in the regression line, is very close to the experimental result. In case of 3% relative error, both experimental and simulation show that the displacement of the specimen should be more than 2 pixel.

Figure 8 shows that 8 pixel displacement has a relative error less than 1%. Thus, the practical recommendation from the simulated DIC has been verified. By setting the load to the specimen in order to get displacement on image more than 8 pixel, those practical questions on the introduction section can be eliminated.

Conclusion

In this work, the evaluation of sub-pixel accuracy characteristics has been conducted. It is found that higher subset-template ratio ($R_{st}$) gives a better accuracy. Subset-template ratio between 0.4 and 0.6 should be used in order to prevent undersampling and oversampling of the speckle. Average speckle radius should be kept as minimum as possible to achieve a better accuracy. For 2D-DIC, it is recommended to have displacement on image more than 8 pixel in order to achieve 3% relative error.
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References


