Kinematic Design of Three Degrees of Freedom Planar Parallel Mechanism with Consideration of Workingspace, Singularity and Dexterity

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Abstract

In this paper was studied kinematic design of a three degrees of freedom (dof) planar mechanism. The mechanism was desired to be applied as an assembling equipment composed of three identical kinematic chains connected platform as an output motion and base as a fixed link. The kinematic chains were constructed by three revolute joints (RRR). The platform of mechanism performs three dof planar motion consisting of two dof translational motion and one dof rotational motion. Kinematic constants of the mechanism was determined based on workingspace, singularity and dexterity. There are four kinematic constants of the mechanism consisting of link lengths of kinematic chain, L_1, L_2, distance between mounting two joints in the base, s_b and distance between mounting of kinematic chain to platform and center of platform, r_P. To simplify, the kinematic chain was designed identically and all kinematic constants was normalized with respect to s_b.

It was proposed three synthesis steps to obtained set of kinematic constants. They are evaluation of available workingspace, determination of effective workingspace which is free from singular point and evaluation of dexterity indicating the ability of platform to change its orientation in specific position. Based the work, it was obtained the optimal set of kinematic constants represented by $L_1 = L_2 = 0.3s_b$, $r_P = 0.2r_B$ and $s_b = 1$ unit. Using the such as configuration of kinematic constants, 3-RRR planar parallel mechanism can move translationally in area 0.124 square unit without considering the change of its orientation called as an effective workingspace. Then the platform can change its orentation, 40 degree in area 50% of effective workingspace.

Key words : Kinematic design, dimensional synthesis, planar parallel mechanism, workingspace, singularity, dexterity.

Introduction

Planar parallel mechanism generally is composed of several kinematic chains connected fixed link (base) to output motion link (platform). For three degree of freedom (dof) planar parallel mechanism platform and base is connected by three kinematic chains. Based on such as configuration, the mechanisms provide high rigidity relative to their inertia of moving components beside they can perform stability in high velocity and acceleration of platform motion.

Concerning the performance of planar parallel mechanisms, there are many cases in mechanical system can be substituted by parallel mechanism. High speed and accuracy assembling process is one of possibly of their application. In this work platform of mechanism is required to perform two dof translational in plane and one dof rotational motion. It was proposed some structures of kinematic chains of planar parallel mechanism performing three dof using two basic joints, prismatic (P) and revolute (R) [1]. 3- RRR is one of well known configuration where it is only configured by revolute joint which is applicable as an active and a passive joints.

The problem to realize the parallel mechanism is to yield good performance in workingspace, stiffness and dexterity. This fact relates to complexity on determination of its kinematic constants. Many researchers and practitioners working in parallel mechanism proposed some methods to obtain optimal set of kinematic constants of planar parallel mechanism considering some indices [2-4]. However such as proposed method is limited for special configuration and depend on desired application.

The problem existing of singular point in the workingspace also paid much attention by researchers [5-7]. Some methods to maximize the workingspace which is free from singular point were suggested. In other point of view work in planar parallel mechanism is displacement analysis. Both of forward and inverse kinematic method were developed in order to obtain relationship between input and output displacement [1],[8-9]. The inverse displacement method is more applicable for parallel mechanism because its nature where the mechanism composed of closed loop mechanism. The solution equation of motion can be directly determined.

In this paper was developed kinematic design of 3-RRR planar parallel mechanism. It was proposed steps of synthesis to determine kinematic constants of the mechanism with consideration of workingspace, singularity and dexterity.

Mechanism Architecture

Kinematic diagram of 3-RRR planar parallel mechanism is shown in Fig. 1. The mechanism is composed of base, three identical kinematic chains and platform arranged symmetrically to base and platform. There are four kinematics constants of mechanism represented by, distance between mounting of kinematic chain to base, $s_{b,}$, length of two links in the kinematic chains, L_1, L_2 and radius of platform, r_P.

Using such as notation, length of the kinematic chain can be indicated by vector L_1, L_2 and r_P as shown as Fig. 2. The workingspace of mechanism resulted by intersection area that can be achieved by center of platform, P as depicted in Fig. 3.

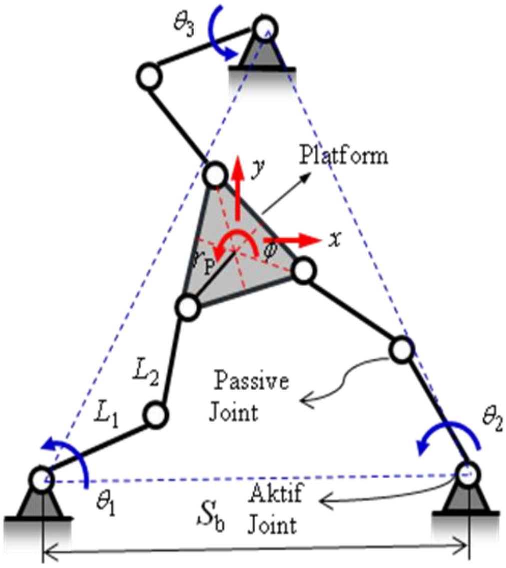

Figure 1 Kinematic Digram 3-RRR Planar Mechanism

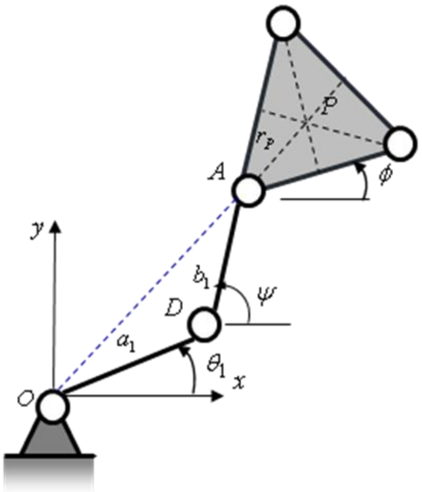

Figure 2 Kinematic chain 3-RRR Planar Mechanism

Using the symmetry arrangement, workingspace mechanism can be calculated using equation Eq. (1),

$$A = \frac{\sqrt{3}}{4}c^2 + 3\left(r^2 \arcsin\frac{c}{2r} - \frac{c}{4}\sqrt{4r^2 - c^2} \right) \quad (1)$$

where c is distance between mounting of kinematic chain to platform.

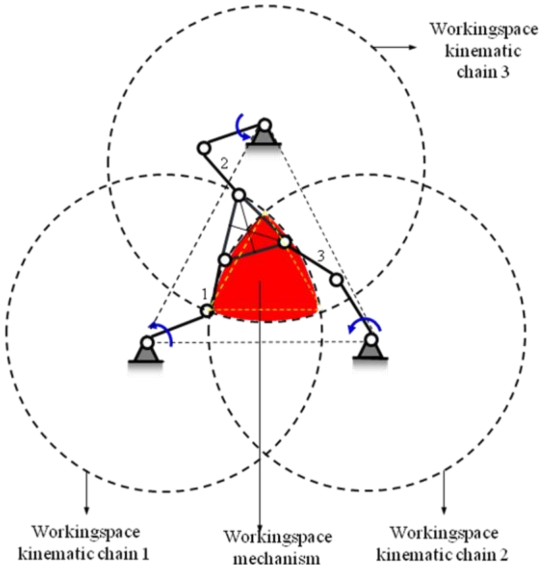

Figure 3 Workingspace 3-RRR mechanism

Displacement Analysis

Using notation of kinematic chain in Fig. 2, displacement analysis of mechanisms can be derived by specifiying position point P with respect to reference coordinate. The vector position point P can be defined as

$$\overline{OP} = \overline{OD} + \overline{DA} + \overline{AP}$$

$$x_P = a_1 \cos(\theta_1) + b_1 \cos(\psi_1) + r_P \cos\left(\frac{\pi}{6} + \phi\right) \quad (1)$$

$$y_P = a_1 \sin(\theta_1) + b_1 \sin(\psi_1) + r_P \sin\left(\frac{\pi}{6} + \phi\right)$$

Relationships between input and output displacement can be expressed by Eq. (2). By eliminating passive joint displacement, ψ_1, Eq. (2) can written as

$$\begin{aligned} & x_P^2 - 2x_p a_1 \cos(\theta_1) - x_P r_P \sqrt{3} \cos(\phi) + x_P r_P \sin(\phi) \\ & + a_1 \cos(\theta_1) r_P \sqrt{3} \cos(\phi) - a_1 \cos(\theta_1) r_P \sin(\phi) + r_P^2 + y_P^2 \\ & - 2 y_P a_1 \sin(\theta_1) - y_P r_P \cos(\phi) - y_P r_P \sqrt{3} \sin(\phi) + a_1^2 \\ & + a_1 \sin(\theta_1) r_P \cos(\phi) + a_1 \sin(\theta_1) r_P \sqrt{3} \sin(\phi) - b_1^2 = 0 \end{aligned} \quad (2)$$

Eq. (3) can be applied for inverse kinematic. In the inverse kinematic, it was specified position and orientation of platform, x_P, y_P, ϕ than input displacement, θ_1, θ_2, θ_3 are calculated .

Singularity Analysis

Generally, in planar parallel mechanisms exist two kinds of singularities where at this condition displacement input do not affect motion of platform. They are serial singularity which usually occurs at boundary of workingspace and parallel singularity which appears in workingspace where at condition platform can move although the tree actuator are locked. In the Fig. 4 and 5 are illustrated the two kinds of the singularities. In Fig. 4 can be seen that one of kinematic chain is fully stretch represented by A_1-B_1-C_1. At this pose, center of platform stay on boundary workingspace.

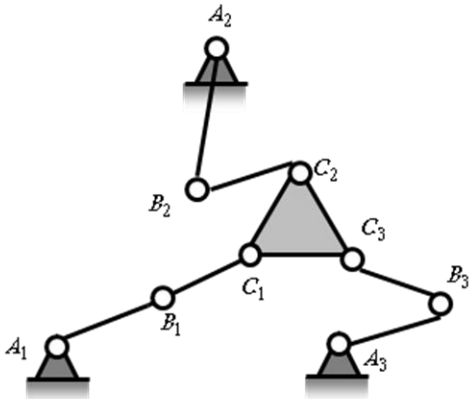

Figure 4 Srial singularity Planar Mechanism

On the other hand, the condition parallel singularity is shown in Fig. 5. The singularity exists when working line of three external forces acting on platform intersect in one point. Although three actuators locked, the platform can rotated freely in the point, I. Line B_i-C_i (i=1,2,3) is line of internal force acting on the platform

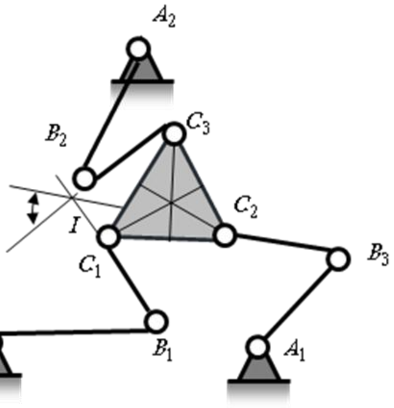

Figure 5 Condition parallel singularity

The singularity condition can be derive analytically by taking velocity transmission by three kinematic chains to the platform. The relationship between input velocity and output velocity can expressed as

$$J_x \dot{x} = J_q \dot{q}$$

where

$\dot{\boldsymbol{x}}$, $\dot{\boldsymbol{q}}$ represente input and output velocity respectivelly,

$$J_x = \begin{bmatrix} b_{1x} & b_{1y} & r_{p1x}b_{1y} - r_{p1y}b_{1x} \\ b_{2x} & b_{2y} & r_{p2x}b_{2y} - r_{p2y}b_{2x} \\ b_{3x} & b_{3y} & r_{p3x}b_{3y} - r_{p3y}b_{3x} \end{bmatrix},$$

$$J_q = \begin{bmatrix} a_{1x}b_{1y} - a_{1y}b_{1x} & 0 & 0 \\ 0 & a_{1x}b_{1y} - a_{1y}b_{1x} & 0 \\ 0 & 0 & a_{1x}b_{1y} - a_{1y}b_{1x} \end{bmatrix},$$

$a_{ix}, a_{iy}, b_{ix}, b_{iy}, r_{Pix}, r_{Piy}$ (i=1,2) represented position points A, B and P on the mechanism displayed in Fig.2. The singularity will occur when determinant of matrix J_x and J_q equal to zero.

Methodology of dimensional synthesis

Step of synthesis of 3-RRR planar parallel mechanism was carried out in three steps. The first step of synthesis is evaluation of effective workingspace by considering length of kinematic chains. In this steps will be decided the length optimal of kinematic chain giving large percentage of effective workingspace defined as wokringspace which is free from singular point.

The second steps is determination of composition of kinematic constants, L_1, L_2, r_P, s_b. Decision set kinemtic constant will be influenced area of workingspace. For simplification, $L_1 = L_2$, and all the kinematic constants normalized with respect to s_b. The last step is to evaluate achievement of dexterity of platform to change it orientation in the effective workingspace.

Results and Discussion

The effective workingspace was obtained for various length of kinematic constants represented by length of vector composed of L_1, L_2 and r_P which is normalized to s_b . It was obtained relationship between percentage area of effective workingspace with respect to available workingspace indicated the workingspace that can be achieved by the platform without concerning singular point and length of kinematic chain as shown in Table 1. Base on result, it was obtained that the optimal length of kinematic chain is 0.8 with area of effective workingspace equal to 98% of available wokingspace.

In next step of synthesis is to optimize configuration of length of Composition of L_1, L_2 and r_P to yield 0.8 length of kinematic chain. In this work, length of link was equal for simplification. Using this constraint, it is obtained the optimal configuration dimension of link length, and radius of platform as plotted in Table 2. Based on the result, it was obtained that the optimal configuration dimension of kinematic constants are $L_1 = L_1$= 0.3 and r_P = 0.2 which can provide area of effective workingspace equal to 98% of available wokingspace.

The final evaluation regarding the achievement of mechanism is dexterity. In this step of work it is obtained relationships between area of wokingspace with respect to capability of platform to rotate in specific position. Based on the result shown in Table 2 the area of effective workingspace is 98% without concerning rotation capability or the platform was desired to move pure translational motion. In Table 3 was displayed achieving of percentage of effective workingspace relates to range of ration angle of platform, ϕ. Based on the result it is shown that platform can rotate in large angle in the limited workingspace. This fact shown interference between kinematic chains and limitation of length of kinematic chains.

Table 1 Relationship between length of kinematic chain and percentage of effective workingspace

Length of Kinematic Chain (normalize to s_b)	0.6	0.7	0.8	0.9	1
Percentage of effective workingspace (%)	72	95	98	90	78

Table 2 Relationship between radius of platform and percentage of effective workingspace

Radius of platform (normalize to s_b)	0.1	0.2	0.3	0.4	0.5	0.6
Percentage of effective workingspace (%)	95	98	94	69	37	35

Table 3 Relationship between dexterity and percentage of effective workingspace

Dexterity (degree)	20	30	40	50	60	70
Percentage of effective workingspace (%)	62	55	35	30	3	0

Using the dimension obtained in the synthesis, in Fig. 6 was shown the Cad design of mechanism. Based on design two link on the kinematic chain are identical to reduce complexity in manufacturing and assembling. The platform of mechanism can move translationally in area 0.124 square unit without change it orientation.

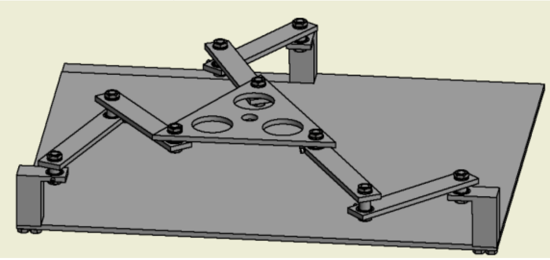

Fig. 6 Cad diagram of 3-RRR mechanism

Conclusions

Based on work in kinematic synthesis of 3-RRR planar parallel mechanism, It was obtained optimal set of kinematic constant , link lengths L_1=L_2 = 0.3 and radius of platform r_P = 0.2 which normalized to distance between mounting of kinematic chain to base, s_b. Using such configuration the platform have 50% of effective workingspace effective if platform is employed rotate maximum 40° and platform can perform pure translation motion in 0.124 square unit area of effective workingspace.

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