Analytical Study on Onset of Convection in a Porous Medium

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Abstract

The onset of convection in a porous medium has been studied analytically. A two-dimensional hypothetical geothermal system is utilized as the model. Applying the Nusselt number and Rayleigh number for which the onset of convection generates, the calculated results show that for the considered system, the onset of convection is attained when the system is subjected to heat source of 0.25 W/m^2 which corresponds to the temperature at the base of 115^{o}C. Furthermore, the heat source subjected at the base of the system and the cell width of the model are varied in the range of 0.3 – 8 W/m^2 and 200 – 3000 m respectively, to study the effects of subjected heat source and cell width on the temperature difference between upper and lower boundaries.

Keywords: onset of convection, porous medium, heat source, cell width

Introduction

A number of researches have been carried out on porous media. The application areas of this field can be insulation for buildings and equipments, energy storage and recovery, nuclear waste disposal, chemical reactor and geothermal reservoir. This paper will focus on geothermal application.

In geothermal application, the analytical study of convection in porous media is as part of geothermal reservoir modeling. The geothermal reservoir modeling, basically, can be divided into; reservoir modeling at natural state condition and at production stage. From the natural state modeling we will understand about the estimation of power potential, pressure and temperature distribution in the reservoir, the properties of porous medium, etc. While from the production stage modeling, it can be found the information on the pressure and temperature changes, the scaling problems, mass flow rate change, etc. This paper discusses the natural state modeling.

The aim of this study is mainly to investigate the onset of convection in a porous medium due to various heat sources and cell widths of the system. The problem of onset of convection is considerably important because it is one of the problems involving hydrodynamic instability. The onset of convection is the transition between conductive and convective conditions. The situation where the motionless state of the saturating fluid exists is the conductive condition. It is characterized by the uniform temperature in the entire system. On the other hand, when the temperature of the saturating fluid is not uniform, some flows induced by buoyancy effects will occur. The onset of convection is required to study because it is one of the important phenomena in the geothermal application. It will help us to decide which locations that must be drilled for production wells.

Development of Model

The model of the considered system is shown in figure 1. The system consists of a porous medium filled with fluid. The system is bounded at the bottom of $z=0$ and at the top of $z=H$. The upper boundary of the system is maintained at a constant pressure of P_o and a constant temperature of T_o. The bottom boundary is subjected to a constant heat flux. Both sidewalls of the system are thermally and hydrodynamically insulated. In a geothermal context, P_o and T_o are the atmospheric pressure and temperature, respectively.

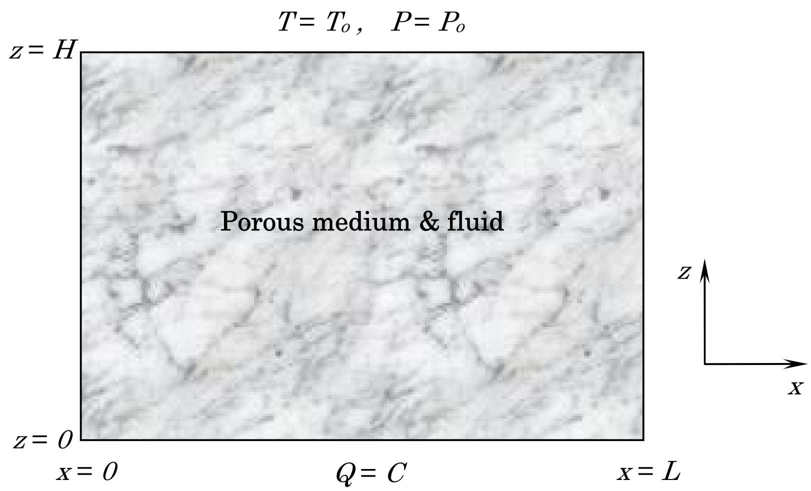

Figure 1. Schematic diagram of the model.

The assumptions for the model are as follows,

- The solid matrix is homogeneous and isotropic
- The solid matrix is non-deformable
- The solid matrix properties are constant
- No heat sources and sinks in the fluid; thermal radiation and viscous dissipation are negligible.

Governing Equations

The governing equations are primarily used to calculate the minimum value of heat source Q or bottom temperature T_1 for which the first convection phenomenon is recognized. The calculations are then performed for other combinations between heat source Q and cell width L.

There are many unknowns for unique solution to be found in this problem. One way of obtaining a closed set of equations may be solved uniquely is to add the constraint that the system is operating in the mode for which the Nusselt number (*Nu*) is a maximum. Therefore, when the system is convecting in the mode for which *Nu* is a maximum, the equations can be expressed as follows,

$$Nu^* = \frac{Q}{k\Delta T / H} \tag{1}$$

$$Ra^* = \frac{Ra}{4\pi^2} = \frac{\rho_o gc\beta KH\Delta T}{4\pi^2 \nu k} \tag{2}$$

$$L^* = \frac{L}{H} \tag{3}$$

where,

Nu/Nu^*	= Nusselt number	ρ_o	= fluid density
Q	= averaged vertical heat flux	g	= gravitational acceleration
k	= thermal conductivity	c	= medium specific heat
ΔT	= temperature difference	β	= coefficient of thermal expansion
Ra/Ra^*	= Rayleigh number	K	= medium permeability
H	= thickness of cell	ν	= fluid kinematic viscosity
L/L^*	= cell width		

A new parameter, namely *C* needs to be defined to integrate the parameters Nu^*, Ra^*, and L^*. The relationship of all above parameters is as follows,

$$Ra^*.Nu^*.(L^*)^2 = C \tag{4}$$

$$C = \frac{\rho_o gc\beta K}{4\pi^2 \nu k^2} QL^2 \tag{5}$$

Utilizing all above equations and solving for $C = 1$, the onset of convection can be evaluated for various combinations of heat source and cell width.

Results and Discussion

The minimum value of heat source *Q* for which the system is in the first convective condition can be calculated by using Eqs. (1), (2), and (3). The typical values for physical properties of medium (rock) and fluid (water) are used to be,

T_o	= 15°C	c	= 1000 J/kg°C
ρ_o	= 1000 kg/m^3	K	= 10^{-14} m^2

β	= 0.001/°C	k	= 2.5 W/m°C
ν	= 10^{-7} m^2/s	g	= 9.8 m/s^2

A sample of calculation below demonstrates how to determine the minimum Q for a given cell width of the system for which the onset of convection occurs. It is clear from the Eqs. (1) and (2), the onset of convection occurs when Nu^* =1 and Ra^* =1. Combining these equations with Eq. (4) for C=1 suggests that the value for L^*=L/H=1. For a given cell width L, say 1000 m, then we get value for H = 1000 m. Substitution of all physical properties and known parameters into Eq. (2) gives ΔT = 100.7°C or the temperature at the bottom is about 115°C. Finally, by utilizing Eq. (1) the minimum heat source Q to be subjected into the system to produce onset of convection is about 0.25 W/m^2.

The effects of cell size on the temperature difference between upper and lower boundaries of the system needs to be investigated. This is important because, for instance, in a geothermal system the extent of the geothermal system is defined by boundaries (hot spots) which, in this case, corresponds to twice of the cell width. By knowing the measured cell width L and the average surface heat flux Q, we can estimate the depth and temperature of the source. Utilizing Eq. (5) gives C. Having calculated C, we can calculate L^* and Ra^*. Finally, we can obtain the depth of the system H and the bottom temperature $T_o + \Delta T$ from Eqs. (3) and (2), respectively. By using the previous data for typical physical properties of rock and water, the results of calculation is presented in Table 1.

Table 1. Convective system parameters for a typical set of rock and water properties

Q (W/m^2)	**L (m)**	**C**	**Ra^***	**H (m)**	**ΔT (°C)**	**$T^o + \Delta T$ (°C)**
0.3	1000	1.2	1.05	1053	100.4	115.4
	2000	4.8	2.37	2381	100.2	115.2
	3000	10.7	5.17	5660	92.0	107.0
3	400	1.9	1.26	425	298.6	313.6
	600	4.3	2.10	674	313.8	328.8
	800	7.6	3.89	1127	347.6	362.6
8	200	1.3	1.05	206	513.3	528.3
	300	2.8	1.55	316	494.0	509.0
	400	5.1	2.47	690	360.5	375.5

The procedure of finding the temperature difference ΔT and the heat source Q using the above analysis is only applied for the critical condition as indicated by the parameters in the equations (Ra^* and Nu^*). It can be seen that the equations represent the condition at which the system is in the last conductive process. It means that at the same depth the temperature of the system is uniform. For the given system, we cannot analyze the condition at which the process is in conduction or if the subjected heat source is less than the minimum calculated heat source. Once we define one parameter, it allows us to find other parameters. For the given system, the ratio between the cell width and depth, $L^* = 1$. It gives $Ra^* = 1$ and $Nu^* = 1$. When the system has conduction mode, then $Nu^* < 1$. The result is that the value of $C < 1$, so that we cannot apply Eq. (4) to find Ra^*. If the heat source Q is greater than the minimum heat source at which the system starts convection, we cannot apply Eqs. (1) to (5). For such conditions, the solution is solved using other method which is beyond the study.

From table 1, it can be seen that the values of ΔT are independent of cell width L and increase approximately proportional to $Q^{1/2}$. It can be explained as follows.

Substitution of H from Eq. (2) into (1) gives,

$$Q = \frac{\rho_o g c \beta K}{4\pi^2 \nu} \frac{Nu^*}{Ra^*} (\Delta T)^2 \tag{6}$$

Giving $L^* = L/H = g(Ra^*)$, then it is clear that L should appear in Eq. (6). Because Eq. (4) suggests that Nu^* is approximately equal to Ra^*, therefore, both parameters will disappear from Eq. (6) and L as well. The equation (6) can be rewritten as,

$$Q \approx \frac{\rho_o g c \beta K}{4\pi^2 \nu} (\Delta T)^2 \tag{7}$$

$$\Delta T \approx a Q^{1/2} \tag{8}$$

where,

$$a = \left[\frac{4\pi^2}{\rho_o g c \beta K} \right]^{1/2} \tag{9}$$

By using Eq. (7), then we can estimate the temperature of the bottom (base) of the system by measuring Q only. Furthermore, the depth of the system can be estimated using Eqs. (1) to (5) after measuring the cell width L which is half distance between hot spots. This is because we consider that the system consists of one cell only.

Conclusions

1. The analytical study on onset of convection for a given system consisting of porous medium filled with fluid can be used to estimate the temperature of the bottom and the depth of the system knowing heat discharge from the system and horizontal extent of the system.
2. The calculated temperature at the bottom of the system is independent of cell width.

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