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Control a Single-Arm Robot Using an Electro-Hydraulic Actuator

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Abstract

In this paper, controlling a rigid robot manipulator driven by hydraulic system is considered. Due to changing of inertia moment of the manipulator as well as the effect of fiction of hydraulic system, the dynamics of the robot arm is highly nonlinear. Therefore sliding mode control method is applied. Sliding mode control is a robust control method. It provides a symmetric approach to the problem of maintaining stability and consistent performance in the face of modeling imprecision. Nummerical simulations of the control system are represented. The results show the extremely good robustness of the proposed method.

Index Terms—Hydraulic robots, Control of robot manipulators, Sliding mode control, Servo-hydraulic control.

INTRODUCTION

Hydraulic robots and machinery are widely used in the construction and mining industries as crane, excavator, robotic [5], [12], [13], [14]... They have rapid responses and high power-to-weight ratios suitable for many applications. Furthermore, the potential complexity of such controllers is becoming less and less of an implementation issue due to the inexpensive and powerful processors available today for real-time control.

Unfortunately, the control of hydraulic manipulators is more challenging that of their electrical counterparts because of the highly nonlinear hydraulic dynamics [8], [9]. Non-linear characteristics originate from the compressibility of the fluid and complex characteristic of servo valve. In addition, significant uncertain nonlinearities such as external disturbance, leakages and friction are unknown and can be not modeled accurately. Therefore the classical control methods as PI, PID… can be not applied effectively to control hydraulic manipulators as hydraulic actuators cannot accurately apply forces or torques over a significant dynamic range. It is very important to find a suitable nonlinear control method to hydraulic manipulators. In this paper, a sliding mode control is applied to control a single rigid manipulator driven by electro-hydraulic system.

Sliding mode control (SMC) or also called variable structure, is a nonlinear control method, which provides a symmetric approach to the problem of maintaining stability and consistent performance in the face of modeling imprecision and disturbances [1],[2],[3],[7]. Many papers used this method to control and showed good results with the effects of un-modeled parameters. In [2], sliding mode control is applied to control a flexible load. To enhance control performance, sliding mode control combined with fuzzy PI controller [6]. But most of them used a symmetric cylinder and so the dynamic of system became simpler as well as controlling. When an asymmetric cylinder is applied, dynamics of the system becomes more complex and using sliding mode control with only a feedback of their position is more difficult. So a new approach of sliding mode control is used. In this way, both errors of position and load pressure is feed backed to design a controller [2], [3]. This method so far just applied for applications of straight motions, not for manipulators.

In this paper, the method originates from [2], [3], and is applied to control a single rigid manipulator. Angle of the manipulator is tracked by following a desired reference angle. Here, pressure between two chambers of cylinder is combined to create a load pressure error of load pressure. Errors of load pressure and angle position are applied to design a controller. Numerical simulation is presented and gives a good tracking.

The rest of the paper is organized as: section II introduces dynamic formulation and problem statement, Controller design is shown in section III. Section IV shows the simulation results and section V concludes the paper.

Dynamic formulation and problem statement

This paper focuses on the single rigid arm control driven by hydraulic system. The coordinate systems, joint angles and physical parameters of the system are defined as in Fig. 1. The kinetic (T) and potential (U) energy of the arm can be determined as [10],

$$T = \frac{1}{2}(ML^2 + \frac{mL^2}{3})\dot{\theta}^2 = \frac{1}{2}(M + \frac{m}{3})L^2\dot{\theta}^2$$
$$U = g(M + \frac{m}{2})L\sin(\theta) \tag{1}$$

Applying the Lagrange's method, the equation of motion are obtained as

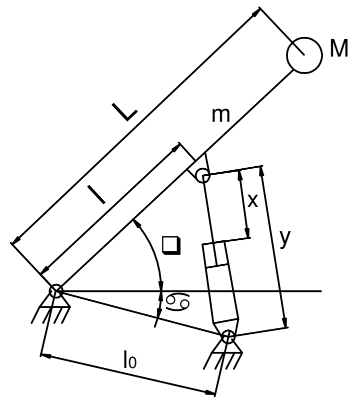

Figure 1: Schematic of the single rigid arm

$$\frac{d}{dt}(\frac{\partial T}{\partial \dot{\theta}}) - \frac{\partial T}{\partial \theta} + \frac{\partial U}{\partial \theta} = R_\theta \tag{2}$$

where R_θ is the generalized force corresponding to the generalized coordinate θ.

$$R_0 = \tau_f - T_f(\dot{\theta}) \tag{3}$$

where $T_f(\dot{\theta})$ is the torque due to friction, is expressed into [3].

$$T_f(\dot{\theta}) = b_1 sign(\dot{\theta}) + b_2\dot{\theta} \tag{4}$$

with b_1 and b_2 are coefficient.

τ_f is the torque of the hydraulic actuator force F on the arm.

$$\tau_f = F\frac{\partial y}{\partial \theta} \tag{5}$$

with y is a new variable and is calculated from Fig.1

$$y = \sqrt{l^2 + l_0^2 - 2ll_0\cos(\theta+\alpha)} \tag{6}$$

and the force F that is applied by the cylinder to the arm is simply

$$F = P_1A_1 - P_2A_2 \tag{7}$$

where P_1 and P_2 are the head and rod end pressure of the cylinder, A_1 and A_2 are the head and rod areas of the cylinder respectively.

Substituting Eqs. (1), (3)-(7) into Eqs.(2), gives the equation of the single rigid arm.

$$(M+\frac{m}{3})L^2\ddot{\theta} + (M+\frac{m}{2})gL\cos\theta + T_f(\dot{\theta}) = (P_1A_1 - P_2A_2)\frac{\partial y}{\partial \theta} \tag{8}$$

The schematic of the hydraulic servo system driving the single rigid manipulator is showed in Fig.2. Here P_s and P_r are the supply pressure and the tank reference pressure.

For simplicity, a first-order equation of servo valve is represented in this study [12]. The spool valve displacement x_v is related to the current input I by

$$\tau_v x_v = -x_v + K_I I \tag{9}$$

where τ_v and K_v are the time constant and gain of the servo valve respectively.

The amount of fluid flow to the head-side Q_1 and from the rod-side Q_2 of the cylinder is a function of both the valve spool position and cylinder pressures. The relationship can be expressed in the following form [8]:

$$\begin{aligned} Q_1 &= k_q g_1(P_1, sign(x_v))x_v \\ Q_2 &= k_q g_2(P_2, sign(x_v))x_v \end{aligned} \tag{10}$$

where k_q is the flow gain coefficient of the servo valve, g_1 and g_2 are functions of x_v and P_1, P_2.

$$\begin{aligned} g_1(P_1, sign(x_v)) &= \begin{cases} \sqrt{P_s - P_1}, & x_v \ge 0 \\ \sqrt{P_1 - P_r}, & x_v < 0 \end{cases} \\ g_2(P_2, sign(x_v)) &= \begin{cases} \sqrt{P_2 - P_r}, & x_v \ge 0 \\ \sqrt{P_s - P_2}, & x_v < 0 \end{cases} \end{aligned} \tag{11}$$

Applying the flowing continuity equations to the two sides of the cylinder and neglecting any external leakage [8]:

$$\begin{aligned} Q_1 &= A_1\frac{dy}{dt} + \frac{V_0 + A_1x}{\beta_e}\dot{P}_1 + C_{tm}(P_1 - P_2) \\ Q_2 &= A_2\frac{dy}{dt} - \frac{V_0 + A_2(L-x)}{\beta_e}\dot{P}_2 + C_{tm}(P_1 - P_2) \end{aligned} \tag{12}$$

where β_e is the effective bulk modulus, C_{tm} is the coefficient of the internal leakage of the cylinder, V_0 is the dead volume of the fluid inside each chamber of the cylinder, x is the displacement of the cylinder and is a function of y.

Define a state vector for the system as follows:

$$x \overset{\Delta}{=} [x_1\ x_2\ x_3\ x_4]^T = [\theta\ \dot{\theta}\ P_1\ P_2]^T \tag{13}$$

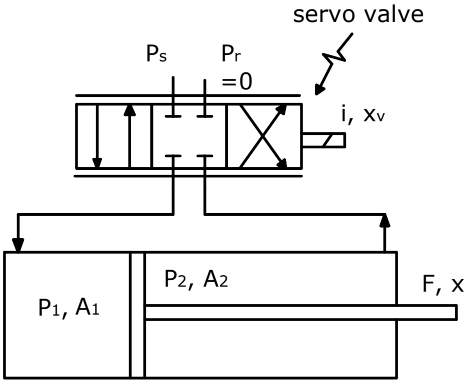

Figure 2: Hydraulic servo system

By combining (8)-(13), gives the following nonlinear state-space model of the system:

$$\dot{x}_1 = x_2 \quad (14a)$$

$$\dot{x}_2 = a_{21}\cos x_1 + a_{23}x_3 + a_{24}x_4 - d \quad (14b)$$

$$\dot{x}_3 = a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + b_1 I \quad (14c)$$

$$\dot{x}_4 = a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + b_2 I \quad (14d)$$

where

$$C_1 = (M + \frac{m}{3})L^2 \quad , \quad C_2 = (M + \frac{m}{2})gL$$

$$a_{21} = -\frac{C_2}{C_1}; \quad a_{23} = \frac{A_1}{C_1}\frac{\partial y}{\partial \theta}; \quad a_{24} = -\frac{A_2}{C_1}\frac{\partial y}{\partial \theta}; \quad d(t) = \frac{T_f(\dot{\theta})}{C_1} \quad (15)$$

$$a_{32} = -\frac{\beta_e A_1}{V_1}\frac{\partial y}{\partial \theta}; \quad a_{33} = -\frac{\beta_e C_{tm}}{V_1}; \; a_{34} = +\frac{\beta_e C_{tm}}{V_1}; \; b_1 = \frac{\beta_e kqg_1}{V_1}\frac{K_I}{\tau s + 1}$$

$$a_{42} = \frac{\beta_e A_2}{V_2}\frac{\partial y}{\partial \theta}; \quad a_{43} = \frac{\beta_e C_{tm}}{V_2}; \; a_{44} = -\frac{\beta_e C_{tm}}{V_2}; \; b_2 = -\frac{\beta_e kqk_v g_2}{V_2}\frac{K_I}{\tau s + 1}$$

We define a load pressure as

$$\hat{x}_3 = P_1 - \frac{A_2}{A_1}P_2 \quad (16)$$

So new state vector of the system is $x = \left[x_1 \; x_2 \; \hat{x}_3\right]^T$. Now we state the control problem of this system follows. A controller has to be designed such that the operating state vector $x = \left[x_1 \; x_2 \; \hat{x}_3\right]^T$ tracks asymptotically a desired state vector $x_d = \left[x_{1d} \; x_{2d} \; \hat{x}_{3d}\right]^T$ which is predetermined with a finite control input I in the presence of uncertainty of the system.

Sliding mode controller design

1. Design for switching surface

From equation (14), on realizes that the dynamics the system is highly nonlinear. To enhance the system performance, the following surface is defined [2],[3]:

$$s = e_c + \lambda e_c, \; \lambda > 0 \quad (17)$$

where e_c is the combination of position error and load pressure error described as follows:

$$e_c = w_1 e_1 + w_2 \int e_1 dt + w_3 \int e_3 dt, \quad w_1, w_2 \geq 0, w_3 > 0 \quad (18)$$

where

$$e_1 = x_1 - x_{1d}$$
$$e_3 = \hat{x}_3 - \hat{x}_{3d} \tag{19}$$

Control block diagram is shown in Figs. 3. x_1, x_3 and x_4 are sent back by sensors to design a controller to track following the desired input x_{1d}.

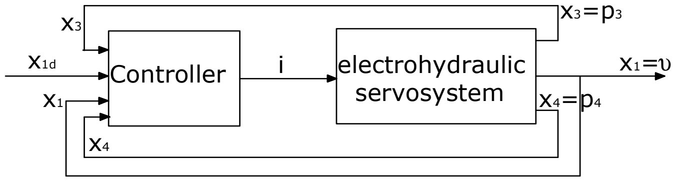

Figure 3: Control block diagram

Taking the derivative of e_1 and e_3, gives

$$\begin{aligned} &\dot{e}_1 = \dot{x}_1 - \dot{x}_{1d} \\ &\dot{e}_2 = \dot{x}_2 - \dot{x}_{2d} = a_{21}\cos x_1 + a_{23}x_3 + a_{24}x_4 - d - \dot{x}_{1d} \\ &\dot{e}_3 = \dot{\hat{x}}_3 - \dot{\hat{x}}_{3d} = (a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + b_1 u) - \frac{A_2}{A_1}(a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + b_2 u) - \dot{\hat{x}}_{3d} \\ &\dot{e}_3 = (a_{32} - \frac{A_2}{A_1}a_{42})x_2 + (a_{33} - \frac{A_2}{A_1}a_{43})x_3 + (a_{34} - \frac{A_2}{A_1}a_{44})x_4 + bu - \dot{\hat{x}}_{3d} \end{aligned} \tag{20}$$

with

$$b = b_1 - \frac{A_2}{A_1}b_2 \tag{21}$$

The assumptions of this method are described as follows:

1) $a_{ij} = \hat{a}_{ij} + \Delta a_{ij},\ b = \hat{b} + \Delta b,\ i = 2,3,4,\ j = 1,\ 2,\ 3,4.$
   $\left|\Delta a_{ij}\right| \le \alpha_{ij},\ \ |\Delta b| \le \beta,\ \forall t$
2) $|d| \le \gamma$
3) $\{x_{1d},\ \dot{x}_{1d}, \ddot{x}_{1d},\ \hat{x}_{3d}, \dot{\hat{x}}_{3d}\}$ are bounded and known

where $\hat{a}_{ij}$ and $\hat{b}$ are calculated by the nominal parameters of the system. α_{ij}, β and γ, the upper bound of the parameter uncertainties are known.

The desired position and differential pressure and their derivatives are previously available (or assigned). The desired pressured $\hat{x}_{3d}$ is assumed to be:

$$\hat{x}_{3d} = \frac{1}{A_1}\frac{\partial x_{1d}}{\partial y}\left[C_1\ddot{x}_{1d} + C_2\cos x_{1d} + f_c C_1 \gamma\right] \tag{22}$$

Where $f_c \in [0,1]$ which is introduced to average the peak error of e_1.

2. Sliding mode controller
a. Theory

Before developing the proposed controller, the following analysis of sliding surface dynamics of equation (17) is given. One sufficient condition for the asymptotic performance of the dynamical system of equation (17)-(19) is achieved in the following theorem:

Theorem: If the dynamics of a sliding surface satisfy the condition $s \to 0$ for $t > t_1$, then $e_c \to 0$ as $t \to \infty$.

Proof: First, the following Lyapunov's function is defined:

$$V_s(e_c) = 0.5e_c^2 > 0, \quad e_c \neq 0 \tag{23}$$

Taking the time derivative of $V_s(e_c)$ and substituting $\dot{e}_c$ in Eqs. (17) and $s \to 0$ for $t > t_1$, gives

$$\dot{V}_s(e_c) = e_c\dot{e}_c = e_c[s - \lambda e_c] = -\lambda e_c^2 < 0 \text{, for } t > t_1 \tag{24}$$

Because $\dot{V}_s(e_c)$ for $t > t_1$, e_c is bounded for $t > t_1$, then from Eqs (18), e_1 and e_3 are bounded for $t > t_1$. From the assumptions 1-3, $\dot{e}_1(t)$ is bounded for $t > t_1$.Using Eqs. (18),

$$e_c = w_1e_1 + w_2e_1 + w_3e_3 \tag{25}$$

Because $\dot{e}_1, e_1$ and e_3 are all bounded, $\dot{e}_c$ is bounded $\forall t > t_1$. Hence, by applying Barbalat's lemma [7], $e_c \to 0$ as $t \to \infty$.

b. Design the control law

The control law

$$I = I_{eq} + I_{sw} \tag{26}$$

The equivalent control law

$$\begin{aligned} I_{eq} = -\frac{1}{\hat{b}w_3}[& w_1(\hat{a}_{21}\cos x_1 + \hat{a}_{23}x_3 + \hat{a}_{24}x_4 - \ddot{x}_{1d}) \\ & + w_2e_2 \\ & + w_3\left((\hat{a}_{32} - \frac{A_2}{A_1}\hat{a}_{42})x_2 + (\hat{a}_{33} - \frac{A_2}{A_1}\hat{a}_{43})x_3 + (\hat{a}_{34} - \frac{A_2}{A_1}\hat{a}_{44})x_4 - \dot{\hat{x}}_{3d}) \right) \\ & + \lambda(w_1e_1 + w_2e_2 + w_3e_3)] \end{aligned} \tag{27a}$$

and a discontinuous is added to (27a)

$$I_{sw} = -\frac{1}{\hat{b}w_3}K sign(S) \tag{27b}$$

c. Stability Analysis

Assume that a Lyapunov's function is described as follows:

$$V(s) = \frac{1}{2}s^2 > 0, \quad \text{for } s \neq 0 \tag{28}$$

To achieve perfect tracking, all system trajectories have to converge to s in finite time and stay on s afterwards, the condition is

$$\dot{V}(s) < 0. \tag{29}$$

Taking the time derivative of $V(s)$ of Eqs.(28) gives

$$\dot{V}(s) = s\dot{s} \tag{30}$$

Combining with Eqs (17)-(27) gives

$$\dot{V}(s) = s.\ [w_1(a_{21}\cos x_1 + a_{23}x_3 + a_{24}x_4 - d - \ddot{x}_{1d}) + w_2 e_2 + w_3\left((a_{32} - \frac{A_2}{A_1}a_{42})x_2 + (a_{33} - \frac{A_2}{A_1}a_{43})x_3 + (a_{34} - \frac{A_2}{A_1}a_{44})x_4 + bI - \dot{\hat{x}}_{3d})\right) + \lambda(w_1 e_1 + w_2 e_2 + w_3 e_3)] \quad (31)$$

Substituting the control law Eqs.(27) into Eqs (31) gives:

$$\dot{V}(s) = s.\ [w_1(\Delta a_{21}\cos x_1 + \Delta a_{23}x_3 + \Delta a_{24}x_4 - d) + w_3\left((\Delta a_{32} - \frac{A_2}{A_1}\Delta a_{42})x_2 + (\Delta a_{33} - \frac{A_2}{A_1}\Delta a_{43})x_3 + (\Delta a_{34} - \frac{A_2}{A_1}\Delta a_{44})x_4 + \Delta b I_{eq}\right) - \frac{\hat{b} + \Delta b}{\hat{b}} K sign(s)] \quad (32)$$

Because $\frac{\hat{b} + \Delta b}{\hat{b}} > 1$, Eqs.(32) gives

$$\dot{V}(s) \le |s|[F - K sign(s)] \quad (33)$$

where

$$F = w_1(\Delta\alpha_{21}\cos x_1 + \alpha_{23}x_3 + \alpha_{24}x_4 + \gamma) + w_3\left((\alpha_{32} - \frac{A_2}{A_1}\alpha_{42})x_2 + (\alpha_{33} - \frac{A_2}{A_1}\alpha_{43})x_3 + (\alpha_{34} - \frac{A_2}{A_1}\alpha_{44})x_4 + \beta I_{eq})\right) \quad (34)$$

By choosing the gain K

$$K = F + \eta \quad , \eta > 0 \quad (35)$$

The inequality (33) becomes

$$\dot{V}(s) \le -\eta|s| \ \text{, so} \ \dot{V}(s) \le 0 \quad (36)$$

Therefore $s \to 0$ for any $t > t_1$ and from the theorem above, $e_c \to 0$ as $t \to \infty$.

Numerical Simulation

1. Simulation

An electro-hydraulic servo-system with the nominal parameters: L = 2m, l = 0.98m, l_0 = 0.85m, α = 100^0, m = 20kg, M = 22kg, b_1 = 2Nm, b_2 = -3Nmrad^{-1}s, Ps = 315e5Pa, β_e = 15e9, A_1 = 0.002m^2, A_2 = 0.001m^2, V_0 = 5e-4m^3, L_0 = 1m, C_{tm}=2e-12m^{3}s^{-1}Pa, k_q=1.5370e-5, K_I = 5e-4, $\tau = 0.0025$ is simulated by the fourth-order Runge-Kutta method with $T_c = 0.005s$ time interval. The initial value of state is $x[0] = [15^0\ 0\ 200e5\ 300e5]^T$. Figs. 4-7 show the results which the system tracks the reference step input response $x_{1d} = 30^0$. The results with a reference sinusoidal input x_{1d} =22.5+7.5sin(0.5πt) are shown in Fig. 8-11.

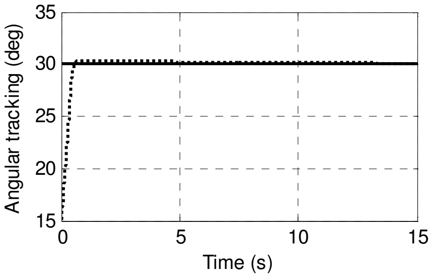

Figure 4: Position tracking

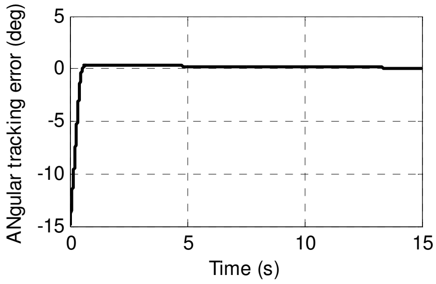

Figure 5: Position tracking error

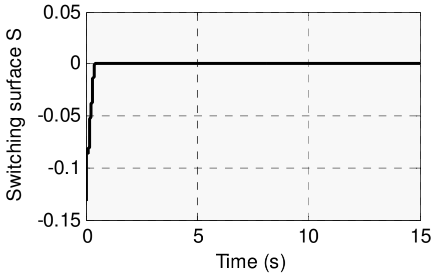

Figure 6: Switching function S(x,t)

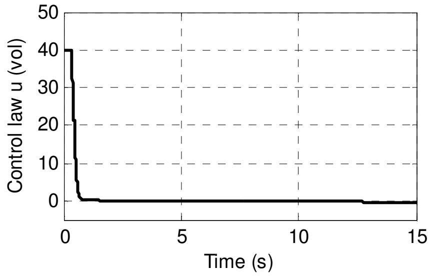

Figure 7: Control input u

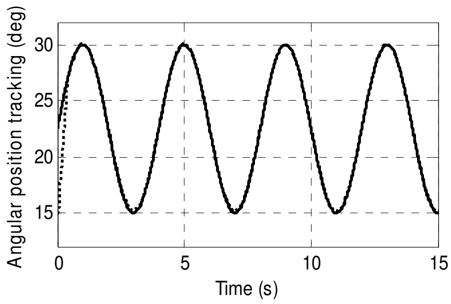

Figure 8: Position tracking

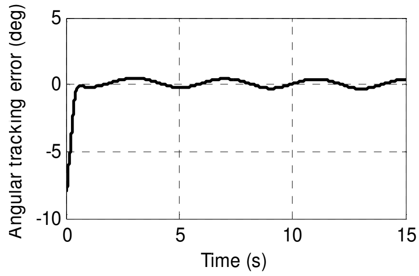

Figure 9: Position tracking error

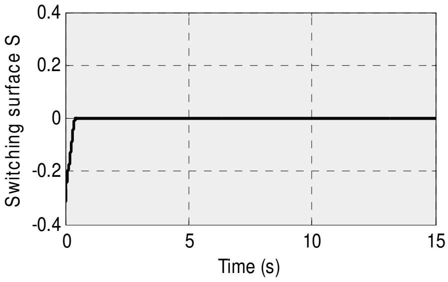

Figure 10: Switching function S(x,t)

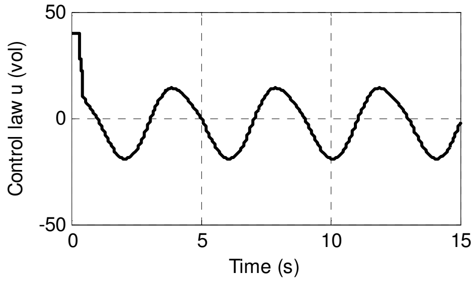

Figure 11: Control input u

2. Discussion

The good tracking results are achieved for both step and sinusoidal inputs (see Figs. 4 and 8). Even with uncertain parameters, the angular position of the electro-hydraulic servo-system can asymptotically track the desired time-varying trajectory. Its angular position maximum error is between -0.02^0 to 0.02^0 (see Figs.5 and (9). The operating point of system is convergent to the neighbourhood of the sliding surface (see Figs. 6 and 10). Control inputs for two cases are quite smooth enough (see Figs.7-11).

Conclusion

A sliding mode control is applied to a rigid manipulator. The combination of angular position error and the load pressure error can be asymptotically tracked even when the system is subject to parameter uncertainties. Numerical simulation results have shown good performances of tracking.

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