ACCELERATED ANISOTROPIC ROTOR THROUGH ITS CRITICAL SPEEDS

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ABSTRACT

This research deals a non-stationary anisotropic rotor with different shaft orientation through its bending critical speeds. In case of an anisotropic rotor has the difference in the shaft orientation, in which the direction of the principal axis of the shaft cross-section in the left shaft end is different from the direction in the right shaft end. The effect of the gyroscopic moments must be taken into account, whether a rigid disk is attached symmetry or asymmetry on the shaft. According to the previous researches, it is well known that the amplitude of the unbalance response of a rotor which runs through a critical speed can be reduced by increasing the value of the acceleration. The anisotropic rotor model with different shaft orientation is run up until through the critical speeds. The dynamic responses of the rotor models are compared and depicted for various anisotropic coefficients and differences in the shaft orientations. The higher the anisotropy coefficient of the rotor, the higher is the maximum amplitude. For the rotor with the same element anisotropy, but the difference in the shaft orientation $\Delta\beta$ *is varied, the bigger the difference in the shaft orientation, the lower is the reached maximum amplitude.*

Keywords: anisotropic rotor, shaft orientation, run up operation

1. Introduction

It is well known that the amplitude of the unbalance response of a rotor which runs through a critical speed can be reduced by increasing the value of the acceleration. Iwatsubo et al [1] studied the non-stationary vibration of an asymmetric rotor passing through its critical speed. In their models, two approaches have been used. In the first approach, the system is driven at constant acceleration, for which an energy source provides an ideal driving force to the vibrating system. The second one is an energy source interacting with the vibrating system, where the system is driven by a non-ideal energy source. Markert et al [2] investigated a minimal torque that is needed to accelerate an elastic rotor to pass the first bending critical speed. Meanwhile, Markert found that as reported in [3] and [4], the maximum rotor deflection is smaller than during stationary resonance speed. The maximum rotor deflection does not appear when the rotor speed corresponds to the critical speed. The peak is shifted to a higher frequency during run-up and shifted to a lower frequency during run-down. After running through resonance the vibrational components at natural frequency dominate but will decay with time. Ganesan [5] analyzed the effect of bearings and shaft asymmetry on the stability of the rotor. Particular attention has been paid to the motion characteristics of the rotor while passing through the primary resonance. The presence of proper combination between bearings and shaft asymmetries on the rotor helps the stability of the unbalance response during start-up or run-down operation.

In order to describe the complete nature of the problem, additional characteristics must be also considered. In this problem, the Jeffcott rotor is no longer a satisfactory model. Therefore, several solutions are proposed by using an approach of a discrete or continuous rotor. Gasch et al [6] and Markert [7] investigated a flexible rotor with a continuous mass distribution passing through its critical speeds under a driving torque. Similar to the authors above, Genta and Delprete [8] approached a rotor system with multiple degrees of freedom by using the finite element method. However, none of the researchers above studied about anisotropic rotor with different shaft orientation. A published paper which investigated anisotropic rotor with different shaft orientation is introduced by Malta [9]. In that paper, the rotor is approached by using the minimal number of discrete model. The effects of the gyroscopic moments which come from the difference in the shaft orientation and the asymmetry position of the disk on the shaft were studied. In the analyses, the rotor stability is considered only at constant angular velocity. In the present work, the similar model with the model in the previous paper [9] will be conducted at constant angular acceleration (i.e. run up operation).

2. System Modelling

In Figure 1, the rotor shaft is modelled by two discrete elements which have different orientations of the principal axes of the element cross-sections. The rotor is simply supported by two rigid bearings. In order to simplify the shaft anisotropy, a rectangular cross section of the shaft is used. The anisotropies of all shaft elements are the same. By using the minimal number of elements, the rotor is discretized into two elements with the length ℓ_1 and ℓ_2 , respectively. Therefore, the mathematical model can be simplified and possesses only four degrees of freedom, where the vibration of the shaft comes from only the motions ($\zeta_W, \eta_W, \varphi_\zeta, \varphi_\eta$) of the disk. The disk is assumed to be a thin disk. In the whole system, the internal and external dampings are taken into account. As a point of interest, although the disk is attached symmetrically on the shaft ($\ell_1 = \ell_2$) and all elements have the same cross-sectional moment of inertia, but because of different orientation between the shaft elements, the disk position makes a precession. In this case, the effect of gyroscopic moments is no longer negligible.

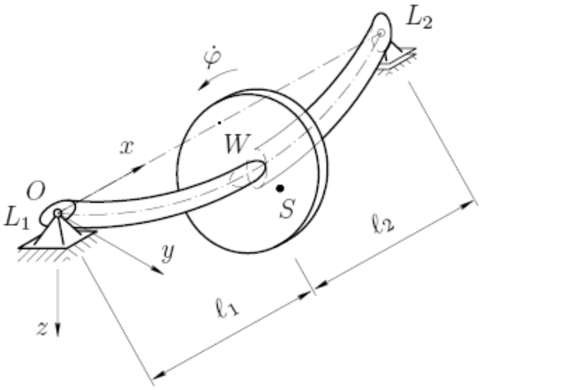


Figure 1 Anisotropic rotor with difference shaft orientation supported by rigid bearings

Based on the Fig. 2, if the rotor is assumed that has the minimal number of discrete elements (i.e. shaft with two elements only) the subscribe *k* has a value 1 or 2. Furthermore, the coordinate systems of the principal axes of the first and the second shaft element are placed on the η_1^* - ζ_1^* - plane inclined at an angle β_1 and the η_2^* - ζ_2^* - plane at angle β_2 . The centre of gravity *S* of the disk is eccentric to the centre of the shaft *W* and its position being defined by the eccentricity ε and the angular position ϕ .

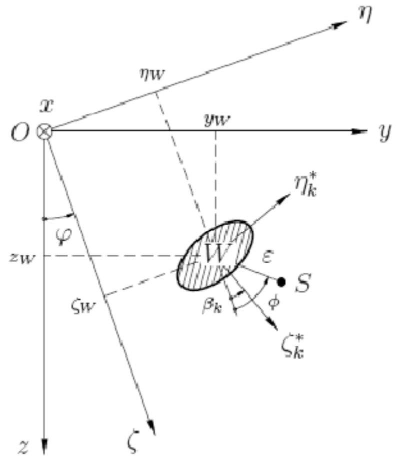


Figure 2 Coordinate system of anisotropic rotor

In rotating reference frame, the moments of inertia of cross-section in principal axes of shaft elements are obtained by using the following equations

$$I_{k\zeta} = \int \eta_k^2 \, dA = \tfrac{1}{2}\left(I_{k\eta^*} + I_{k\zeta^*}\right) - \tfrac{1}{2}\left(I_{k\eta^*} - I_{k\zeta^*}\right)\cos 2\beta_k - I_{k\eta^*\zeta^*}\sin 2\beta_k \quad (1a)$$

$$I_{k\eta} = \int \zeta_k^2 \, dA = \tfrac{1}{2}\left(I_{k\eta^*} + I_{k\zeta^*}\right) + \tfrac{1}{2}\left(I_{k\eta^*} - I_{k\zeta^*}\right)\cos 2\beta_k + I_{k\eta^*\zeta^*}\sin 2\beta_k \quad (1b)$$

$$I_{k\eta\zeta} = -\int \eta_k \zeta_k \, dA = -\tfrac{1}{2}\left(I_{k\eta^*} - I_{k\zeta^*}\right)\sin 2\beta_k + I_{k\eta^*\zeta^*}\cos 2\beta_k \quad (1c)$$

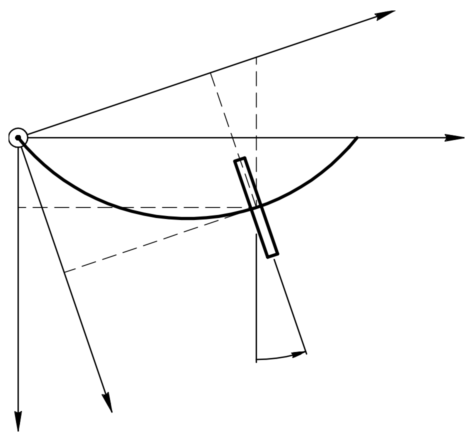
Furthermore, as shown in Fig. 3, the disk on the shaft is described in the coordinate system (*x′, y′, z′*), where the plane of disk is parallel to the *y′*-*z′*-plane. The *x′*-axis is perpendicular to that plane. Furthermore, *y′*-axis can move only in the *x*-*y*-plane and *z′*-axis in the *x*-*z*-plane, therefore y'-axis and z'-axis can be not-perpendicular, where their position can make precessions φ_z and φ_y , respectively. This means the coordinate system (*x′, y′, z′*) is no longer orthonormal.

From the Fig. 3, the transformation equations of basis vectors are obtained

$$\begin{aligned} \vec{e}_{z'} &= \sin\varphi_y \, \vec{e}_x + \cos\varphi_y \, \vec{e}_z \\ \vec{e}_{y'} &= -\sin\varphi_z \, \vec{e}_x + \cos\varphi_z \, \vec{e}_z \\ \vec{e}_{x'} &= \frac{\vec{e}_{y'} \times \vec{e}_z{}'}{\left|\vec{e}_{y'} \times \vec{e}_z{}'\right|} = \frac{\vec{e}_x + \tan\varphi_z \, \vec{e}_y - \tan\varphi_y \, \vec{e}_z}{\sqrt{1 + \tan^2\varphi_y + \tan^2\varphi_z}} \; . \end{aligned} \quad (2)$$

Further, the kinematics relationships of angular velocities in (x', y', z')-coordinate system are determined. If angular speed of the disk is denoted by ω_S in the (x', y', z')-coordinate system, the y'-z'-plane that rotates along the x'-axis is denoted by ω_E and $\dot{\varphi}$ is the rotational speed of the shaft, then

$$\omega_S = \omega_E - \dot{\varphi}\,\vec{e}_{x'} \ . \tag{3}$$



(a)

(b)

Figure 3 Coordinate of disk in anisotropic rotor system

Furthermore, angular speed of basis vectors $\vec{e}_{y'}$ and $\vec{e}_{z'}$ are

$$\omega(\vec{e}_{y'}) = \dot{\varphi}_{x'}(\vec{e}_{y'})\vec{e}_{x'} + \dot{\varphi}_{y'}(\vec{e}_{y'})\vec{e}_{y'} + \dot{\varphi}_{z'}(\vec{e}_{y'})\vec{e}_{z'} \tag{4}$$

and

$$\omega(\vec{e}_{z'}) = \dot{\varphi}_{x'}(\vec{e}_{z'})\vec{e}_{x'} + \dot{\varphi}_{y'}(\vec{e}_{z'})\vec{e}_{y'} + \dot{\varphi}_{z'}(\vec{e}_{z'})\vec{e}_{z'} , \tag{5}$$

respectively. Note that, the expression in parenthesis is not a function argument but an alternative index. For an example, the $\dot{\varphi}_{x'}(\vec{e}_{y'})\vec{e}_{x'}$ means the rotational speed of the vector $\vec{e}_{x'}$ due to y'-axis. Because the plane of disk is placed at the y'-z'-plane and the precession φ_z is the angle of the plane of disk with respect to the z-axis, hence

$$\omega(\vec{e}_{y'}) = \dot{\varphi}_z\,\vec{e}_z \,. \tag{6}$$

Similar to the Eq. (6), the precession φ_y is the angle of the plane of disk with respect to the y-axis, hence

$$\omega(\vec{e}_{z'}) = \dot{\varphi}_y\,\vec{e}_y \,. \tag{7}$$

By using the Cramer's rule, angular speed $\dot{\varphi}_{x'}(\vec{e}_{y'})$, $\dot{\varphi}_{y'}(\vec{e}_{y'})$, $\dot{\varphi}_{z'}(\vec{e}_{y'})$, $\dot{\varphi}_{x'}(\vec{e}_{z'})$, $\dot{\varphi}_{y'}(\vec{e}_{z'})$ and $\dot{\varphi}_{z'}(\vec{e}_{z'})$ of the basis vectors can be determined. Based on the Figure 3, it is clear that the angular speed of the y'-z'-plane is the rotational speed of the vector $\vec{e}_{y'}$ due to z'-axis and the rotational speed of the vector $\vec{e}_{z'}$ due to y'-axis, hence the angular speed in Eq. (3) can be reformulated as

$$\omega_E = \dot{\varphi}_{y'}(\vec{e}_{z'})\vec{e}_{y'} + \dot{\varphi}_{z'}(\vec{e}_{y'})\vec{e}_{z'} \ . \tag{8}$$

By inserting the basis vectors of the results of the Cramer's rule and the Eq. (8) into the Eq. (3), the ω_S can be reformulated. Furthermore, the vector of angular momentum can be calculated

$$L = \Theta\,\omega_S \ . \tag{9}$$

If the precessions φ_z and φ_y are assumed to be small then

$$\begin{aligned} L = &\left(-\Theta_p\dot{\varphi}\right)\vec{e}_x + \left(-\Theta_p\dot{\varphi}\varphi_z + \Theta_a\dot{\varphi}_y\right)\vec{e}_y \\ &+ \left(\Theta_p\dot{\varphi}\varphi_y + \Theta_a\dot{\varphi}_z\right)\vec{e}_z \ . \end{aligned} \tag{10}$$

The time derivative of angular momentum in rotating reference frame can be rewritten as

$$\begin{aligned} \frac{dL}{dt} = &\left(-\Theta_p\ddot{\varphi}\right)\vec{e}_x + \left[\Theta_p\left(\ddot{\varphi}\varphi_\eta + \dot{\varphi}^2\varphi_\zeta + \dot{\varphi}\dot{\varphi}_\eta\right)\right. \\ &\left. + \Theta_a\left(\ddot{\varphi}_\zeta - \dot{\varphi}^2\varphi_\zeta - 2\dot{\varphi}\dot{\varphi}_\eta - \ddot{\varphi}\varphi_\eta\right)\right]\vec{e}_\zeta \\ &+ \left[\Theta_p\left(-\ddot{\varphi}\varphi_\zeta + \dot{\varphi}^2\varphi_\eta - \dot{\varphi}\dot{\varphi}_\zeta\right)\right. \\ &\left. + \Theta_a\left(\ddot{\varphi}_\eta - \dot{\varphi}^2\varphi_\eta + 2\dot{\varphi}\dot{\varphi}_\zeta + \ddot{\varphi}\varphi_\zeta\right)\right]\vec{e}_\eta \end{aligned} \tag{11}$$

For the case in Figure 1, the rotor model has a node which has four degrees of freedom, those are two