STABILITY INVESTIGATION OF ANISOTROPIC ROTOR WITH DIFFERENT SHAFT ORIENTATION SUPPORTED BY ANISOTROPIC BEARINGS

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ABSTRACT

This research deals a stability investigation of anisotropic rotor with different shaft orientation which supported by anisotropic flexible bearings. In case of an anisotropic rotor has the difference in the shaft orientation, in which the direction of the principal axis of the shaft cross-section in the left shaft end is different from the direction in the right shaft end. The rotor is approached by using the minimal number of discrete model. The effects of the gyroscopic moments come not only from the difference in the shaft orientation, but also from the asymmetry position of the disk on the shaft and the difference of bearing stiffness. Because the anisotropic rotor is supported by anisotropic flexible bearings, the system stiffness must be a time-variant parameter whether the rotor is modelled in a fixed or in a rotating reference frame. The stability charts of the anisotropic rotor supported by anisotropic flexible bearings are analyzed by using the Floquet's theory. Comparing to the rotor which has the same parameters but is supported by rigid bearings, while the stability chart of the rotor supported by rigid bearings has only a single region of instability in the whole varying coefficients of the element anisotropy, the rotor in flexible bearings has three separated intervals of instabilities at lower values of the element anisotropy.

Keywords: anisotropic rotor, shaft orientation, anisotropic flexible bearings, rotor stability, Floquet's theory

1. Introduction

An anisotropic rotor system can be modelled both in a fixed and in a rotating reference frames. In a fixed reference frame, the dynamic parameter especially the shaft stiffness is time-variant. If the rotor is modelled in a rotating reference frame, where the coordinate system follows the rotation of the shaft, then the differential equations of the system become speed-dependent. Hence, at constant rotational speed, the dynamic parameters of the rotor can be considered constant. If an anisotropic rotor is supported by anisotropic flexible bearings, the system stiffness must be a time-variant parameter whether the rotor is modelled in a fixed or in a rotating reference frame. Thus, the mathematical model is more complicated. The effects of the gyroscopic moments are influenced not only by the difference in the shaft orientation but also from asymmetry position of the disk on the shaft and the difference of bearing stiffness.

According to the previous researches about anisotropic rotor in anisotropic bearings, various papers have been published. Hull [1] conducted experiments and showed the forward and backward whirl motion of anisotropic rotors which are supported by flexible anisotropic bearings. Iwatsubo et al [2] concerned with the vibrations of an asymmetric simple rotor supported by asymmetric bearings. The effects of asymmetry of the rotor and the bearing stiffness have been analyzed. However, some parameters in the model like the eccentricity of the rotor, the acceleration of gravity, the effects of rotary inertia, gyroscopic moments, and shear deformation were neglected.

Furthermore, the stability of rotors modelled by discrete elements has been also investigated. Oncescu et al [3] formulated a set of ordinary differential equations with periodic coefficients by using finite element method in conjunction with a time-transfer matrix method based on Floquet's theory. In their model, the shaft cross-section is asymmetric, having different principal moments of inertia and varied step-by-step along the longitudinal axis. However, the principal directions of inertia of the cross-section are uniform along the shaft. By taking into account of the shear deformation in the rotor model (i.e. Timoshenko beam), Chen and Peng [4] and also Boru and Irretier [5] have analyzed the stability of the rotor based using the finite element method. However, none of the researchers above studied about anisotropic rotor with different shaft orientation. An anisotropic rotor with different shaft orientation has been introduced by Malta [6]. In that paper, the rotor is approached by using the minimal number of discrete model. The effects of the gyroscopic moments come not only from the difference in the shaft orientation, but also from the asymmetry position of the disk on the shaft. In the analyses, the rotor stability is considered at constant angular velocity. However, the rotor model is supported only by rigid bearings, in which the stability areas can be determined directly through the analyses of eigenvalues.

The present research is developed based on the reference [6] for anisotropic rotor case which is supported by anisotropic flexible bearings, in which the system is time-variant. One method which can solve such equation is the Floquet's theory [7], [8], [9].

2. System Modelling

The system is investigated as an anisotropic rotor supported by anisotropic flexible bearings as shown in Fig. 1. In this case, besides the anisotropy in each bearing, it is possible that the deflections between the left and the right bearing are different. Therefore, the effect of gyroscopic moments can be increased or decreased.

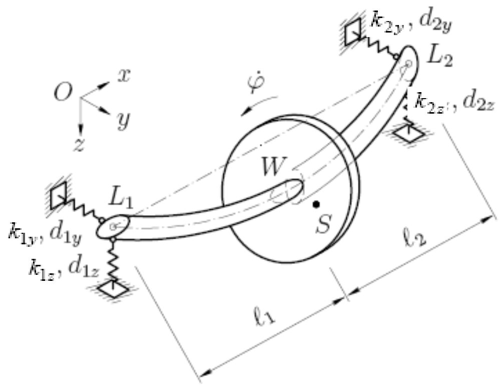

Figure 1 Anisotropic rotor with difference shaft orientation supported by anisotropic flexible bearings

Based on the Fig. 2, if the rotor is assumed that has the minimal number of discrete elements (i.e. shaft with two elements only) the subscribe k has a value 1 or 2. Furthermore, the coordinate systems of the principal axes of the first and the second shaft element are placed on the η_1^* - ζ_1^* - plane inclined at an angle β_1 and the η_2^* - ζ_2^* - plane at angle β_2. The centre of gravity S of the disk is eccentric to the centre of the shaft W and its position being defined by the eccentricity ε and the angular position ϕ. The left and the right shaft ends are denoted as L_1 and L_2, respectively.

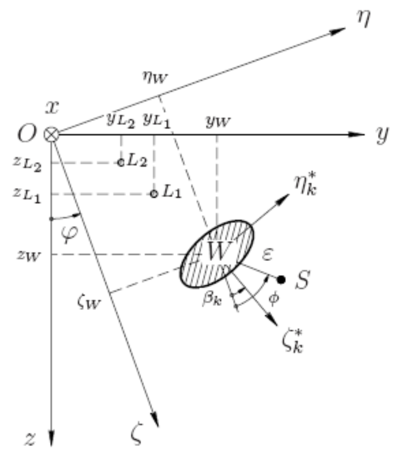

Figure 2 Coordinate system of anisotropic rotor supported by anisotropic flexible bearings

Furthermore, as shown in Fig. 3, the disk on the shaft is described in the coordinate system (x', y', z'), where the plane of disk is parallel to the y'-z'-plane. The x'-axis is perpendicular to that plane. Furthermore, y'-axis can move only in the x-y-plane and z'-axis in the x-z-plane, therefore y'-axis and z'-axis can be not-perpendicular, where their position can make precessions φ_z and φ_y , respectively. This means the coordinate system (x', y', z') is no longer orthonormal. Because the rotor system is supported by anisotropic flexible bearings, the precessions φ_y and φ_z occur not only due to the slope of the shaft at deflection in the left and in the right bearings, but also by slope of the deflected shaft, hence

$$\varphi_y = \varphi_{y_W} + \varphi_{y_L} \quad \text{and} \quad \varphi_z = \varphi_{z_W} + \varphi_{z_L} \tag{1}$$

where φ_{y_W} and φ_{z_W} are the slopes of the disk due to the axis of the shaft in undeflected condition in the x-z and x-y-plane, respectively (i.e. the precessions are occurred only by shaft deflection). The φ_{y_L} and φ_{z_L} are the precessions that come from the slope of the shaft due to the deflection in the left and the right bearing.

From the Fig. 3, the transformation equations of basis vectors are obtained

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$$\vec{e}_{z'} = \sin\varphi_y \, \vec{e}_x + \cos\varphi_y \, \vec{e}_z$$
$$\vec{e}_{y'} = -\sin\varphi_z \, \vec{e}_x + \cos\varphi_z \, \vec{e}_z \tag{2}$$
$$\vec{e}_{x'} = \frac{\vec{e}_{y'} \times \vec{e}_z{}'}{\left|\vec{e}_{y'} \times \vec{e}_z{}'\right|} = \frac{\vec{e}_x + \tan\varphi_z \, \vec{e}_y - \tan\varphi_y \, \vec{e}_z}{\sqrt{1 + \tan^2\varphi_y + \tan^2\varphi_z}} \, .$$

Further, the kinematics relationships of angular velocities in (x', y', z')-coordinate system are determined. If angular speed of the disk is denoted by ω_S in the (x', y', z')-coordinate system, the y'-z'-plane that rotates along the x'-axis is denoted by ω_E and $\dot{\varphi}$ is the rotational speed of the shaft, then

$$\omega_S = \omega_E - \dot{\varphi} \, \vec{e}_{x'} \; . \tag{3}$$

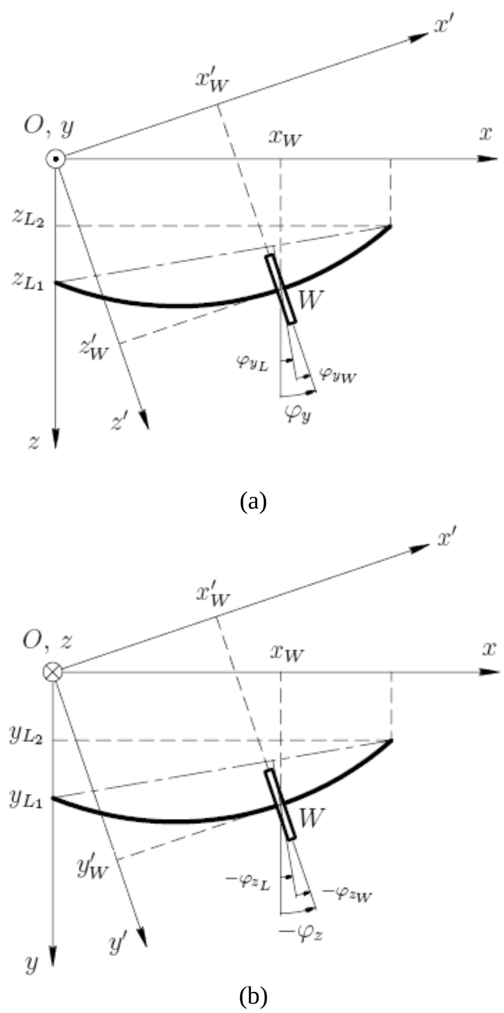

Figure 3 Coordinate of disk in anisotropic rotor-bearings system

Furthermore, angular speed of basis vectors $\vec{e}_{y'}$ and $\vec{e}_{z'}$ are

$$\omega(\vec{e}_{y'}) = \dot{\varphi}_{x'}(\vec{e}_{y'})\vec{e}_{x'} + \dot{\varphi}_{y'}(\vec{e}_{y'})\vec{e}_{y'} + \dot{\varphi}_{z'}(\vec{e}_{y'})\vec{e}_{z'} \tag{4}$$

and

$$\omega(\vec{e}_{z'}) = \dot{\varphi}_{x'}(\vec{e}_{z'})\vec{e}_{x'} + \dot{\varphi}_{y'}(\vec{e}_{z'})\vec{e}_{y'} + \dot{\varphi}_{z'}(\vec{e}_{z'})\vec{e}_{z'} \, , \tag{5}$$

respectively. Note that, the expression in parenthesis is not a function argument but an alternative index. For an example, the $\dot{\varphi}_{x'}(\vec{e}_{y'})\vec{e}_{x'}$ means the rotational speed of the vector $\vec{e}_{x'}$ due to y'-axis. Because the plane of disk is placed at the y'-z'-plane and the precession φ_z is the angle of the plane of disk with respect to the z-axis, hence

$$\omega(\vec{e}_{y'}) = \dot{\varphi}_z \, \vec{e}_z \, . \tag{6}$$

Similar to the Eq. (6), the precession φ_y is the angle of the plane of disk with respect to the y-axis, hence

$$\omega(\vec{e}_{z'}) = \dot{\varphi}_y \, \vec{e}_y \, . \tag{7}$$

By using the Cramer's rule, angular speed $\dot{\varphi}_{x'}(\vec{e}_{y'})$, $\dot{\varphi}_{y'}(\vec{e}_{y'})$, $\dot{\varphi}_{z'}(\vec{e}_{y'})$, $\dot{\varphi}_{x'}(\vec{e}_{z'})$, $\dot{\varphi}_{y'}(\vec{e}_{z'})$ and $\dot{\varphi}_{z'}(\vec{e}_{z'})$ of the basis vectors can be determined. Based on the Figure 3, it is clear that the angular speed of the y'-z'-plane is the rotational speed of the vector $\vec{e}_{y'}$ due to z'-axis and the rotational speed of the vector $\vec{e}_{z'}$ due to y'-axis, hence the angular speed in Eq. (3) can be reformulated as

$$\omega_E = \dot{\varphi}_{y'}(\vec{e}_{z'})\vec{e}_{y'} + \dot{\varphi}_{z'}(\vec{e}_{y'})\vec{e}_{z'} \; . \tag{8}$$

By inserting the basis vectors of the results of the Cramer's rule and the Eq. (8) into the Eq. (3), the ω_S can be reformulated. Furthermore, the vector of angular momentum can be calculated

$$L = \Theta \, \omega_S \; . \tag{9}$$

If the precessions φ_z and φ_y are assumed to be small then

$$L = \left(-\Theta_p\dot{\varphi}\right)\vec{e}_x + \left(-\Theta_p\dot{\varphi}\varphi_z + \Theta_a\dot{\varphi}_y\right)\vec{e}_y + \left(\Theta_p\dot{\varphi}\varphi_y + \Theta_a\dot{\varphi}_z\right)\vec{e}_z \; . \tag{10}$$

The time derivative of angular momentum in rotating reference frame can be rewritten as

$$
\begin{aligned}
\frac{dL}{dt} = & \left(-\Theta_p \ddot{\varphi}\right) \vec{e}_x + \Big[\Theta_p\left(\ddot{\varphi}\varphi_\eta + \dot{\varphi}^2\varphi_\zeta + \dot{\varphi}\dot{\varphi}_\eta\right) \\
& + \Theta_a\left(\ddot{\varphi}_\zeta - \dot{\varphi}^2\varphi_\zeta - 2\dot{\varphi}\dot{\varphi}_\eta - \ddot{\varphi}\varphi_\eta\right)\Big] \vec{e}_\zeta \\
& + \Big[\Theta_p\left(-\ddot{\varphi}\varphi_\zeta + \dot{\varphi}^2\varphi_\eta - \dot{\varphi}\dot{\varphi}_\zeta\right) \\
& + \Theta_a\left(\ddot{\varphi}_\eta - \dot{\varphi}^2\varphi_\eta + 2\dot{\varphi}\dot{\varphi}_\zeta + \ddot{\varphi}\varphi_\zeta\right)\Big] \vec{e}_\eta
\end{aligned} \tag{11}
$$

3. Dynamic Parameters

In an anisotropic rotor supported by anisotropic flexible bearings, the system has eight degrees of freedom (DoF) as shown in Fig. 4. Four degrees of freedom come from a node, where a disk is attached and the others come from the motions of bearings. Based on the Fig. 3, the kinematic relationship in rotating reference frame can be determined. Therefore, the translational and rotational displacements of the bearings are obtained as follows

$$\zeta_L = \frac{\ell_2}{\ell}\zeta_{L_1} + \frac{\ell_1}{\ell}\zeta_{L_2}, \tag{12}$$

$$\eta_L = \frac{\ell_2}{\ell}\eta_{L_1} + \frac{\ell_1}{\ell}\eta_{L_2}, \tag{13}$$

$$\varphi_{\zeta_L} \approx -\frac{1}{\ell}\left(\eta_{L_1} - \eta_{L_2}\right), \tag{14}$$

$$\varphi_{\eta_L} \approx -\frac{1}{\ell}\left(\zeta_{L_1} - \zeta_{L_2}\right). \tag{15}$$

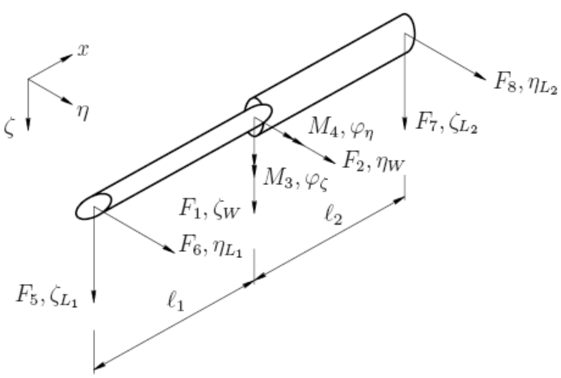

Figure 4 Rotor model 8-DoF with the minimal number of discrete elements

3.1 Flexibility and Damping Matrices

Derivation of equations of the rotor supported by anisotropic flexible bearings is more complicated than the rotor supported by rigid bearings. The differential equations must be considered not only in the shaft system but also in the bearing system. Considering of the shaft stiffness in anisotropic flexible bearings is analogue to the shaft stiffness of the rotor supported by rigid bearings (see Ref. [6]). The internal forces in the shaft are equal to the multiplication of shaft stiffness and relative displacement of the disk, hence

$$
\begin{Bmatrix} F_{k_1} \\ F_{k_2} \\ M_{k_3} \\ M_{k_4} \end{Bmatrix} = -\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{12} & k_{22} & k_{23} & k_{24} \\ k_{13} & k_{23} & k_{33} & k_{34} \\ k_{14} & k_{24} & k_{34} & k_{44} \end{bmatrix} \begin{Bmatrix} \zeta_W - \zeta_L \\ \eta_W - \eta_L \\ \varphi_\zeta - \varphi_{\zeta_L} \\ \varphi_\eta - \varphi_{\eta_L} \end{Bmatrix} \tag{16}
$$

Using Eqs. 12-15, give

$$
\begin{aligned}
\begin{Bmatrix} F_{k_1} \\ F_{k_2} \\ M_{k_3} \\ M_{k_4} \end{Bmatrix} = & -\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{12} & k_{22} & k_{23} & k_{24} \\ k_{13} & k_{23} & k_{33} & k_{34} \\ k_{14} & k_{24} & k_{34} & k_{44} \end{bmatrix} \begin{Bmatrix} \zeta_W \\ \eta_W \\ \varphi_\zeta \\ \varphi_\eta \end{Bmatrix} \\
& + \begin{bmatrix} k_{15} & k_{16} & k_{17} & k_{18} \\ k_{25} & k_{26} & k_{27} & k_{28} \\ k_{35} & k_{36} & k_{37} & k_{38} \\ k_{45} & k_{46} & k_{47} & k_{48} \end{bmatrix} \begin{Bmatrix} \zeta_{L_1} \\ \eta_{L_1} \\ \zeta_{L_2} \\ \eta_{L_2} \end{Bmatrix}
\end{aligned} \tag{17}
$$

or rewrite in simple form

$$\{f_k\} = -[K_W]\{q_W\} + [K_{W_L}]\{q_L\} \tag{18}$$

Analogue to the above shaft stiffness, the internal damping forces can be determined

$$
\begin{Bmatrix} F_{i_1} \\ F_{i_2} \\ M_{i_3} \\ M_{i_4} \end{Bmatrix} = -\begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{12} & d_{22} & d_{23} & d_{24} \\ d_{13} & d_{23} & d_{33} & d_{34} \\ d_{14} & d_{24} & d_{34} & d_{44} \end{bmatrix} \begin{Bmatrix} \dot{\zeta}_W - \dot{\zeta}_L \\ \dot{\eta}_W - \dot{\eta}_L \\ \dot{\varphi}_\zeta - \dot{\varphi}_{\zeta_L} \\ \dot{\varphi}_\eta - \dot{\varphi}_{\eta_L} \end{Bmatrix} \tag{19}
$$

Using the time derivative of Eqs. 12-15, give

$$
\begin{aligned}
\begin{Bmatrix} F_{i_1} \\ F_{i_2} \\ M_{i_3} \\ M_{i_4} \end{Bmatrix} = & -\begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{12} & d_{22} & d_{23} & d_{24} \\ d_{13} & d_{23} & d_{33} & d_{34} \\ d_{14} & d_{24} & d_{34} & d_{44} \end{bmatrix} \begin{Bmatrix} \dot{\zeta}_W \\ \dot{\eta}_W \\ \dot{\varphi}_\zeta \\ \dot{\varphi}_\eta \end{Bmatrix} \\
& + \begin{bmatrix} d_{15} & d_{16} & d_{17} & d_{18} \\ d_{25} & d_{26} & d_{27} & d_{28} \\ d_{35} & d_{36} & d_{37} & d_{38} \\ d_{45} & d_{46} & d_{47} & d_{48} \end{bmatrix} \begin{Bmatrix} \dot{\zeta}_{L_1} \\ \dot{\eta}_{L_1} \\ \dot{\zeta}_{L_2} \\ \dot{\eta}_{L_2} \end{Bmatrix}
\end{aligned} \tag{20}
$$

or rewrite in simple form

$$\{f_i\} = -[D_i]\{\dot{q}_W\} + [D_{i_{LL}}]\{\dot{q}_L\} \tag{21}$$

Because the external damping forces are formulated by proportional damping corresponding to the absolute velocity of the disk, there is no difference of formulation of the equation for anisotropic rotor supported by anisotropic flexible bearings, hence

$$F_a = -d_a\left(\dot{z}_S\,\vec{e}_z + \dot{y}_S\,\vec{e}_y\right) \tag{22}$$

or in rotating reference frame due to centre of shaft *W*, the equation can be formulated as

$$F_a = -d_a\left[\left(\dot{\zeta}_W - \dot{\varphi}\eta_W - \varepsilon\dot{\varphi}\sin\phi\right)\vec{e}_\zeta + \left(\dot{\eta}_W + \dot{\varphi}\zeta_W + \varepsilon\dot{\varphi}\cos\phi\right)\vec{e}_\eta\right] \tag{23}$$

where d_a is coefficient of proportional external damping. In simple matrix form, the Eq. (23) can be rearranged as

$$\{f_a\} = -[D_a]\{\dot{q}_W\} - [K_a]\{q_W\} + \{p_a\}, \tag{24}$$

3.2 Equations in Bearing System

The reaction force for supporting the left side of the rotor in ζ-direction is

$$F_5 = -\left[\frac{\ell_2}{\ell}\left(F_{k_1} + F_{i_1}\right) + \frac{1}{\ell}\left(M_{k_4} + M_{i_4}\right)\right], \tag{25}$$

and in η-direction

$$F_6 = -\left[\frac{\ell_2}{\ell}\left(F_{k_2} + F_{i_2}\right) - \frac{1}{\ell}\left(M_{k_3} + M_{i_3}\right)\right]. \tag{26}$$

The reaction force for supporting the right side of the rotor in ζ-direction is

$$F_7 = -\left[\frac{\ell_1}{\ell}\left(F_{k_1} + F_{i_1}\right) - \frac{1}{\ell}\left(M_{k_4} + M_{i_4}\right)\right] \tag{27}$$

and in η-direction

$$F_8 = -\left[\frac{\ell_1}{\ell}\left(F_{k_2} + F_{i_2}\right) + \frac{1}{\ell}\left(M_{k_3} + M_{i_3}\right)\right]. \tag{28}$$

If a force vector is introduced

$$\{f_{L_W}\} = \left(F_5, F_6, F_7, F_8\right)^T \tag{29}$$

or in simple form of matrix notation

$$\{f_{L_W}\} = [D_{i_L}]^T\{\dot{q}_W\} - [D_{L_L}]\{\dot{q}_L\} + [K_{W_L}]^T\{q_W\} - [K_{L_L}]\{q_L\}. \tag{30}$$

Equations in bearing system are

$$\begin{aligned} F_{L_1} &= -\left[\left(d_{1z}\dot{z}_{L_1} + k_{1z}z_{L_1}\right)\vec{e}_z + \left(d_{1y}\dot{y}_{L_1} + k_{1y}y_{L_1}\right)\vec{e}_y\right] \\ F_{L_2} &= -\left[\left(d_{2z}\dot{z}_{L_2} + k_{2z}z_{L_2}\right)\vec{e}_z + \left(d_{2y}\dot{y}_{L_2} + k_{2y}y_{L_2}\right)\vec{e}_y\right] \end{aligned} \tag{31}$$

where all forces are performed in fixed reference frame, whereas the shaft stiffness is performed in rotating reference frame. Therefore, the equations in the bearings should be transformed into rotating reference frame, hence the equations of the reaction damping forces in the bearings are obtained and written in simple form of matrix notation

$$\{f_{d_L}\} = -[D_{d_L}]\{\dot{q}_L\} - [K_{d_L}]\{q_L\} \tag{32}$$

and

$$\{f_{k_L}\} = -[K_L]\{q_L\} \tag{33}$$

Now, the equations in the bearing system can be determined to

$$[M_L]\{\ddot{q}_L\} = \{f_{L_W}\} + \{f_{d_L}\} + \{f_{k_L}\} \tag{34}$$

3.3 Equations in Rotor System

The differential equations of translatory inertia (i.e. in the ζ and η-directions) in the rotating coordinate system can be determined by using 2nd Newton's Law, hence

$$\begin{aligned} & m\left(\ddot{\zeta}_S - \ddot{\varphi}\eta_S - 2\dot{\varphi}\dot{\eta}_S - \dot{\varphi}^2\zeta_S\right)\vec{e}_\zeta \\ & + m\left(\ddot{\eta}_S + \ddot{\varphi}\zeta_S + 2\dot{\varphi}\dot{\zeta}_S - \dot{\varphi}^2\eta_S\right)\vec{e}_\eta = \\ & \left(F_{a_1} + F_{i_1} + F_{k_1} + F_g\right)\vec{e}_\zeta + \left(F_{a_2} + F_{i_2} + F_{k_2} + F_g\right)\vec{e}_\eta \end{aligned} \tag{35}$$

Note that, the Eq. (35) is still defined in the centre of gravity of the disk. In simple matrix notation, Eq. (35) can be rearranged as

$$[M_T]\{\ddot{q}_W\} + [D_T]\{\dot{q}_W\} + [K_T]\{q_W\} - \{p_T\} = \{f_T\} \tag{36}$$

Furthermore, the differential equations of rotary inertia (i.e. in the φ_ζ and φ_η-directions) can be obtained by the time derivative of angular momentum in rotating reference frame in Eq. (11) and the stiffness matrix of shaft especially in the φ_ζ and φ_η-directions, hence

$$\{M_G\}\{\ddot{q}_W\} + [D_G]\{\dot{q}_W\} + [K_G]\{q_W\} = \{f_G\} \tag{37}$$

Finally, the differential equations of rotor motion can be obtained as follows

$$\begin{aligned}&\left[\begin{array}{c|c}[M_T]+[M_G] & 0\\ \hline 0 & [M_L]\end{array}\right]\begin{Bmatrix}\{\ddot{q}_W\}\\ \{\ddot{q}_L\}\end{Bmatrix}\\ &+\left[\begin{array}{c|c}[D_T]+[D_G]+[D_a]+[D_i] & -[D_{i_L}]\\ \hline -[D_{i_L}]^T & [D_{L_L}]+[D_{d_L}]\end{array}\right]\begin{Bmatrix}\{\dot{q}_W\}\\ \{\dot{q}_L\}\end{Bmatrix}\\ &+\left[\begin{array}{c|c}[K_T]+[K_G]+[K_a]+[K_W] & -[K_{W_L}]\\ \hline -[K_{W_L}]^T & [K_{L_L}]+[K_{d_L}]+[K_L]\end{array}\right]\begin{Bmatrix}\{q_W\}\\ \{q_L\}\end{Bmatrix}\\ &=\begin{Bmatrix}\{p_T\}+\{p_a\}+\{p_g\}\\ 0\end{Bmatrix}\end{aligned} \tag{38}$$

In analysis, the part of $[M_L]$ in Eq. (38) is usually assumped zero, therefore there is a problem if the mass matrix is singular. Because of that, the static condensation method [10] can be used, where the zero part in the diagonal of mass matrix can be eliminated.

In case of a time-variant system, some methods have been developed to solve this problem. One of the widely used methods is Floquet's theory. In this paper, the Floquet's theory is not discussed. For further information about this theory can be found in Ref. [10].

Considering of the time-varying system after Eq. (38) can be conducted numerically, if all parameters in the equations are known. Since the matrix of bearing mass $[M_L]$ is zero, the calculation is also possible by using the static condensation. Furthermore, the equations can be solved by using fourth-order Runge-Kutta method. Therefore, the dynamic responses, especially the steady-state responses (i.e. the responses are assumed after a certain time) can be analyzed in frequency domain for example by using fast fourier transform (FFT). The responses are analyzed for all spin speeds and depicted in a spectral map.

4. Case Study and Discussion

A model of an anisotropic rotor supported by anisotropic flexible bearings is presented. A sketch of the anisotropic rotor model is shown in Fig. 1. The following assumptions are made: the rotor is modelled as a massless shaft and a thin rigid disk is attached in the centre of the shaft ($\ell_1 = \ell_2 = 0.25$ m) and the comparison of the polar mass and axial mass of inertia (Θ_p/Θ_a) is 1.98. In order to simplify the differential equations of the rotor, the shaft is discretized by two discrete elements. Each element has the same dimension (i.e. length and rectangular cross section) but has different shaft orientations. The difference in the shaft orientation is set to $\Delta\beta$. Because of similarity of the resulting stability charts, only the anisotropic rotor with $\Delta\beta = 30^o$ will be presented.

In the numerical simulation, the coefficient of the element anisotropy μ_W is varied from 0 to 0.8. The width *b* of the rectangular cross section is defined to be constant and the thickness *h* of the cross section is formulated as described in the following equation.

$$h = b\sqrt{\frac{1-\mu_W}{1+\mu_W}} \tag{39}$$

The anisotropy in the bearing stiffness is considered by using the following formulations

$$\mu_{L_1} = \left|\frac{k_{1z}-k_{1y}}{k_{1z}+k_{1y}}\right| \quad \text{and} \quad \mu_{L_2} = \left|\frac{k_{2z}-k_{2y}}{k_{2z}+k_{2y}}\right| \tag{40}$$

where k_{1z} and k_{1y} are the stiffness parameters of the bearings on the left shaft end in *z*-direction and *y*-direction, respectively, and k_{2z} and k_{2y} on the right shaft end. The bearing stiffness in the *z*-direction is $k_{1z} = k_{2z} = 13763$ N/m. In this section, the anisotropic rotor with two different anisotropies of the bearing stiffness with $\mu_{L_1} = \mu_{L_2} = 0.3$ (i.e. $k_{1y} = k_{2y} = 7411$ N/m) and 0.6 (i.e. $k_{1y} = k_{2y} = 3441$ N/m) is simulated.

Because the equations of motion of the rotor system are time-variant, the Floquet theory is applied to obtain the stability charts as depicted in Fig. 5 and 6. In the figures, the instability areas are shaded as grey area. Due to numerical restriction, narrow instability tongues between $\Omega/\omega = 2$ and 3 cannot be resolved.

The stability chart of the anisotropic rotor supported by anisotropic flexible bearings as presented in Fig. 5 is compared to the rotor which has the same parameters but is supported by rigid bearings in Ref. [6]. While the stability chart of the rotor supported by rigid bearings has only a single region of instability in the whole varying coefficients of the element anisotropy, the rotor in flexible bearings has three separated intervals of instabilities at lower values of the element anisotropy. The anisotropy coefficient of the bearing stiffness affects the region where the three separated instability intervals emerge to a single interval at higher element anisotropy of the shaft. The higher the anisotropy coefficient of the bearing stiffness, the three separated regions of instabilities reach to higher element anisotropy of the shaft. For the anisotropic rotor with the anisotropic coefficient of the bearing stiffness $\mu_{L_1} = \mu_{L_2} = \mu_L = 0.3$, the three separated regions of instability reach to the element anisotropy $\mu_W < 0.32$ of the shaft. For the same rotor with $\mu_{L_1} = \mu_{L_2} = \mu_L = 0.6$ reaches to $\mu_W < 0.56$.

For reference, the first and the second natural frequencies of the rotor-bearing system are also plotted in Fig. 5 and 6. In this case, the natural frequencies of the system are obtained by solving the characteristic roots of the homogenous differential equations of motion of the rotor according to Eq. (38) after static condensation at rotational speed $\Omega = 0$ and time $t = 0$. At this condition, the first and the second natural frequencies of the system differ. Therefore, this system is unstable at rotational speed in these natural frequencies. Similar to Ref. [9], the second region of the instability at vanishing shaft anisotropy $\mu_W = 0$ is located at $(\omega_1 + \omega_2)/2$.

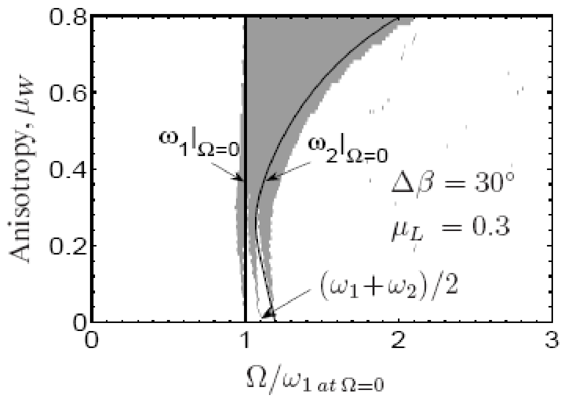

Figure 5 Stability charts according to Floquet of various coefficients of anisotropy ($\mu_W = 0$ to 0.8) of the undamped anisotropic rotor with single disk and the difference $\Delta\beta = 30^o$ in the shaft orientation supported by anisotropic flexible bearings with $\mu_L = 0.3$

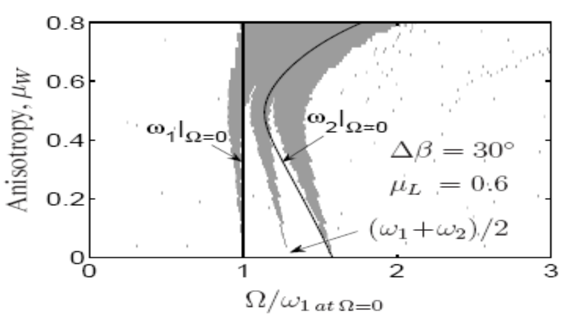

Figure 6 Stability charts according to Floquet of various coefficients of anisotropy ($\mu_W = 0$ to 0.8) of the undamped anisotropic rotor with single disk and the difference $\Delta\beta = 30^o$ in the shaft orientation supported by anisotropic flexible bearings with $\mu_L = 0.6$

Furthermore, the comparisons of dynamic responses in the frequency domain are plotted in Figs. 7-10. The figures show the responses in frequency domain of the rotor for each coefficient of the element anisotropy $\mu_W = 0.2$ and 0.5 and supported by anisotropic flexible bearings with the coefficient $\mu_{L_1} = \mu_{L_2} = \mu_L = 0.3$ (Fig. 7 and 8) and with $\mu_{L_1} = \mu_{L_2} = \mu_L = 0.6$ (Fig. 9 and 10). It is clear that the instability areas occur if the amplitudes of responses are very high either in *z*-direction or in *y*-direction. However, although the amplitudes of the weight critical speed (i.e. at normalized rotational speed about 0.5 - 0.7) are relatively high, but they are not defined as unstable areas in the Floquet stability charts.

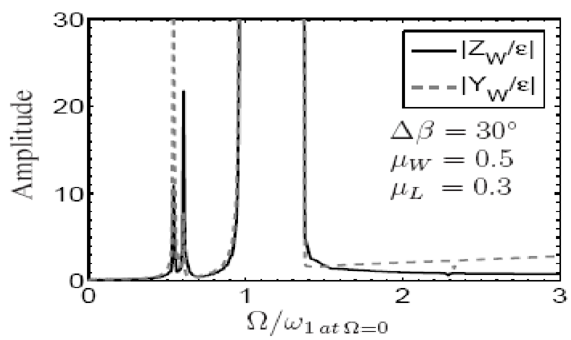

Figure 7 Dynamic responses in frequency domain of the rotor for anisotropy $\mu_W = 0.5$ supported by anisotropic flexible bearings with the coefficient $\mu_L = 0.3$

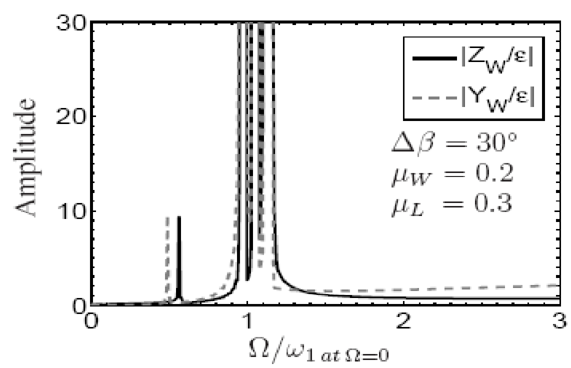

Figure 8 Dynamic responses in frequency domain of the rotor for anisotropy $\mu_W = 0.2$ supported by anisotropic flexible bearings with the coefficient $\mu_L = 0.3$

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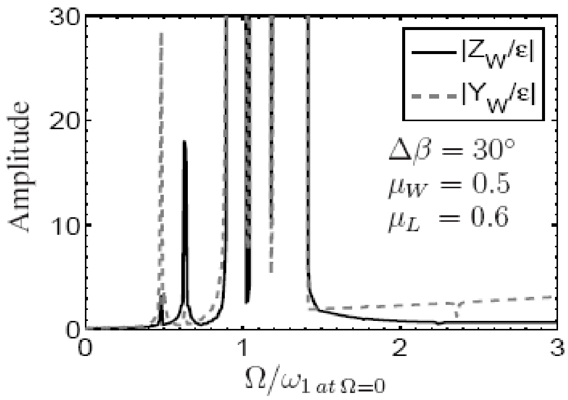

Figure 9 Dynamic responses in frequency domain of the rotor for anisotropy $\mu_W = 0.5$ supported by anisotropic flexible bearings with the coefficient $\mu_L = 0.6$

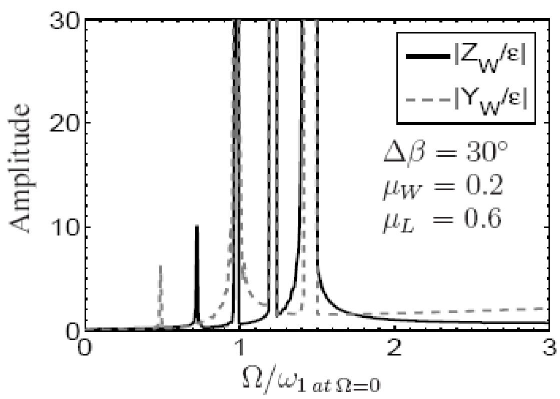

Figure 10 Dynamic responses in frequency domain of the rotor for anisotropy $\mu_W = 0.2$ supported by anisotropic flexible bearings with the coefficient $\mu_L = 0.6$

5. Conclusions

In case of the anisotropic rotor (e.g. with single disk and the shaft is discretized by two elements) supported by anisotropic flexible bearings, the stability chart shows three separated regions of instability especially for the lower element shaft anisotropy. With higher bearings anisotropy, three separated regions of instability reach to a higher element anisotropy of the shaft.

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