Thermoacoustic heat pumping direction change by acoustic field alteration

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Abstract

Thermoacoustic refrigeration is a novel method of cooling utilizing the temperature changes due to pressure oscillations within a sound wave. The absence of hazardous or environmentally harmful refrigerants in a thermoacoustic refrigerator or heater render it as an attractive alternative to conventional vapor compression temperature control methods. The function of a thermoacoustic heat pump can be altered by changing the direction of the acoustic power flow, thus allowing the use of a single device for heating and cooling purposes without additional mechanical equipment. Alteration of the acoustic field can be achieved by changing the phase and/or magnitudes of travelling waves within the thermoacoustic resonator. To this end, we have constructed a thermoacoustic heat pump device using dual opposing acoustic drivers allowing manipulation of negative and positive direction propagating travelling waves. The two acoustic drivers were connected by a resonator tube and a regenerator consisting of layers of steel mesh was positioned in the center. Experiments were conducted using an arbitrary frequency of 260 Hz. Alteration of the acoustic field was achieved by changing the phase difference between the two opposing acoustic drivers. We have tested phase differences from 35^o to 335^o with an acoustic power of 15W. To determine direction of heat pumping within the regenerator, five thermocouples were positioned within. As according to travelling wave thermoacoustics, heat pumping direction flow was opposite the acoustic power flow. At a phase difference of 35^o the temperature difference between thermocouples T_1 and T_2 positioned at the ends of the regenerator was 5 K with T_1 as the hot section and T_2 as the cold section. As the phase difference was increased, the temperature distribution within the regenerator also changed, therefore confirming alteration of heat pumping direction and acoustic power flow

Nomenclature

J_n	*n*th order complex Bessel function
p	pressure
r_0	radius
$r_{h,regen}$	hydraulic radius
Re[ ] , Im[ ]	real and imaginary components
t	time
x	displacement
k	thermal conductivity
α	thermal diffusivity
δ	viscous boundary layer thickness
δ_k	Thermal penetration depth
γ	specific heat ratio
κ	wave number in free space
ρ	density
σ	Prandtl number
τ	thermal relaxation time
ω	angular frequency

Introduction

Thermoacoustic refrigeration is a novel method of temperature control which utilizes fluctuations in temperature caused by pressure oscillations present within sound waves. As this does not require hazardous or environmentally harmful substances, thermoacoustic cooling is an attractive alternative to vapour compression methods which often require ozone layer depleting or global warming associated substances[1]. In a thermoacoustic refrigerator, thermoacoustic heat pumping occurs within a regenerator positioned in the device due to pressure and temperature variations of acoustic waves. The acoustic field that drives the pumping of heat consists of travelling waves propagating in both positive and negative directions and the net acoustic power is influenced by the magnitudes and phasing of the travelling waves.

Early work on thermoacoustic phenomena mainly studied standing wave thermoacoustic effects[2]. As optimum operation of devices utilizing standing wave thermoacoustic effects required a thermal delay, the efficiency of these devices was limited as this introduced irreversibility. To remove this irreversibility, Ceperley proposed to employ travelling wave thermoacoustic effects, instead of standing wave thermoacoustic effects, which do not require a thermal delay thus allowing higher efficiencies[3]. In a travelling wave, the gas undergoes a thermodynamic cycle similar to a Stirling cycle, therefore travelling wave thermoacoustic devices are often referred to as thermoacoustic Stirling devices.

The heart of a thermoacoustic device is its regenerator. The regenerator may consist of stack parallel plates or a series of wire mesh sheets which function is to receive and transfer heat to the oscillating air “packets” of the sound wave. For a travelling wave device, the heat transfer and thermal contact between the gas medium and the regenerator material must be good. Therefore, the distance between parallel plates or the hydraulic diameter must be less than the thermal penetration depth:

$$\delta_k = \sqrt{\frac{k}{\pi f \rho c_p}} \qquad (1)$$

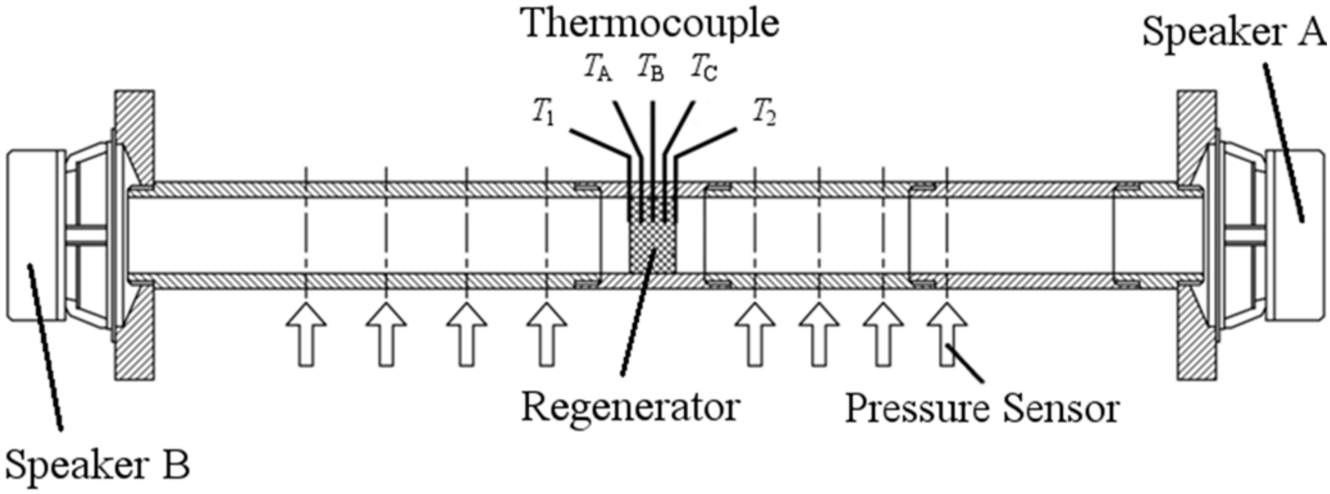

Figure 1 Dual speaker travelling wave theroacoustic device

Therefore, resulting in the temperature of the regenerator at a given point being the same as that of the medium at that point. The continuous heat pumping within the regenerator eventually reveals a temperature distribution corresponding to the direction of heat pumping.

Interestingly, in our previous study regarding a coaxial thermoacoustic refrigerator, we have discovered that the resulting temperature gradient is not always linear and it may also switch signs at a point within the regenerator, i.e. the minimum temperature is within the regenerator[4]. Decomposition of the acoustic field revealed that this was due to the differences of the travelling waves at both ends of the regenerator. The acoustic power flow corresponded to the direction of the dominant travelling wave and this change of dominant travelling wave at a given end of the regenerator will then change the direction of thermoacoustic heat pumping at that end. The acoustic field within the device consists of acoustic travelling waves propagating in positive and negative directions and as such, the regenerator temperature gradient will be influenced by both.

In this study, we experimentally investigated the effect of phase difference between the positive and negative direction travelling waves, and explore the effect of acoustic power within the device using wave decomposition. To investigate the effect of these individual travelling waves, we constructed a thermoacoustic device consisting using two opposite facing speakers thus enabling manipulation of positive and negative direction travelling waves. In addition, by decomposing the measured acoustic field into its positive and negative propagating components and calculating the acoustic power using the two-sensor method, the effect of the individual travelling waves on the direction of acoustic power and regenerator temperature gradient was observed.

Apparatus and Experiment

We constructed a thermoacoustic device consisting of two opposite facing speakers. Altering phase and magnitude of the two speakers independently was possible thus enabling manipulation of positive and negative direction travelling waves. The diagram of dual speaker device is shown in Figure 1. It consisted of an acrylic tube with an inner diameter of 50mm and wall thickness of 5mm. Air at atmospheric pressure and temperature was used as the working fluid within the device. The tube length of 692mm was used corresponding to the half wave length for 250Hz. We have used input electric power of 17W.

The regenerator was constructed of 230 sheets of stainless steel mesh with a mesh number of 150. The mesh sheets were stacked to a total regenerator length of 30mm. The mesh openings had a hydraulic radius of 0.025mm which is smaller than the thermal penetration depth for 250Hz.

The input signals were generated using two Agilent 33220A waveform generators connected to a Yamaha P-1000S Amplifier. Two Jordan JX-92S full range woofers were used as the two opposite facing acoustic drivers. We experimented in changing phase difference from 5º to 335º at intervals of 30º at 250Hz. The thermocouples at the regenerator ends were designated as T_1 located at the end of the regenerator facing speaker B and T_2 located at the end facing speaker A. Three additional thermocouples were positioned within the regenerator as such so that te five thermocouple positions were at regular intervals along the regenerator.

The pressure wave in the tube and the temperature of the regenerator were measured. Temperature measurements were conducted using Type K thermocouples connected to an Agilent 34970A data acquisition unit. The temperatures at the regenerator were recorded for a running time 60 minutes. Pressure measurements were conducted using PCB Piezotronics M102A05 pressure sensors positioned at intervals of 52.5mm and 42mm. The pressure waves at the T_1 end and T_2 end were recorded with a sampling frequency of 50kHz.

Heat exchangers at the ends of the regenerator were not incorporated in the system since they would alter the original temperature distribution within the regenerator caused by the thermoacoustic effect of the positive and negative direction travelling waves. Without the presence of heat exchangers, the change of regenerator temperature distribution in relation to the change of phase difference and magnitudes of the positive and negative direction travelling waves was more clearly observable, later proving important in obtaining the U-shaped temperature distribution described previously.

Thermal contact

The parameter $\omega\tau$ characterizes the thermal contact between the gas medium and the regenerator surfaces[5]. Thermal contact influences the degree of which travelling waves and standing waves influence heat transfer, therefore knowledge of the value of this parameter is important since the observed final thermoacoustic effect is the result of the combination of both the standing wave thermoacoustic effects and the travelling wave thermoacoustic effects[6,7].

The thermal relaxation time τ is calculated by:

$$\tau = \frac{\left(2r_{h,regen}\right)^2}{2\alpha} \qquad (2)$$

A suitable regime for travelling wave operation is obtained when the value of $\omega\tau$ is much smaller than π thus indicating a reversibility of the gas movement within the regenerator channels and gas temperature equating that of the regenerator surfaces. For an operating frequency of 250Hz, a $\omega\tau$ of 0.09 is obtained. Therefore, $\omega\tau$ is much lower than π allowing favorable conditions for travelling wave operation. This value of $\omega\tau$ would render standing wave thermoacoustic effects negligible[7].

Wave decomposition

As standing wave effects are negligible, the resulting temperature gradient within the regenerator will be due to the travelling wave effects. In our previous study, we have observed a sign change of regenerator temperature gradient. By decomposing the combined pressure wave into its positive and negative direction travelling components, we have found that the change in temperature gradient corresponds to the direction of the net acoustic power flow[4]. To evaluate the effect of phase difference on acoustic power flow, here we will also conduct a wave decomposition of the measured pressure wave into its positive and negative direction components at both the T_1 and T_2 sides of the apparatus.

The acoustic pressure wave can be expressed as[8]:

$$w(x,t) = A(\kappa x)\cos\omega t + B(\kappa x)\sin\omega t \qquad (3)$$

where

$$A(\kappa x) = A_1\cos\kappa x + A_2\sin\kappa x \qquad (4)$$

and

$$B(\kappa x) = B_1\cos\kappa x + B_2\sin\kappa x \qquad (5)$$

where $A(\kappa x)$, $B(\kappa x)$ are position-dependent amplitude functions and κ is the wave number in free space, $R(\beta x, \varepsilon t)$ is a residual function having typically a long wavelength characterized by β and a low frequency $\varepsilon << \omega$. As described by Bucher, using the Hilbert transform, the analytic signal describing the relative phase information of the wave is obtained:

$$\begin{aligned} 2w_a(x,t) &= \{(A_1 - B_2) - i(A_2 - B_1)\}e^{i(\omega t+\kappa x)} \\ &+ \{(A_1 + B_2) - i(-A_2 + B_1)\}e^{i(\omega t-\kappa x)} \end{aligned} \qquad (6)$$

The first term and second term of equation are the terms for the travelling waves propagating in the negative (w_-) and positive(w_+) directions respectively. By finding the coefficients A_1, A_2, B_1 and B_2, we can describe the waves separately:

$$2w_-(x,t) = \{(A_1 - B_2) + i(-A_2 - B_1)\}e^{i(\omega t+\kappa x)} \qquad (7)$$

and

$$2w_+(x,t) = \{(A_1 + B_2) + i(A_2 - B_1)\}e^{i(\omega t-\kappa x)} \qquad (8)$$

The change in regenerator temperature gradient involves the change in magnitudes of w_+ and w_- with the dominant travelling wave indicating the direction of the net acoustic power.

Acoustic intensity calculation

According to thermoacoustic theory, heat is pumped opposite to the direction of the propagating travelling wave. Using the two sensor method the acoustic intensity I can be determined for a monofrequency acoustic wave [9,10]:

$$I = \frac{1}{8\omega\rho}\left\{\mathrm{Im}[H](|p_+|^2 - |p_-|^2) + 2\,\mathrm{Re}[H]|p_+||p_-|\sin\theta\right\} \qquad (9)$$

using

$$H = \frac{kF}{\cos(\tilde{k}dx)\sin(kdx)} \qquad (10)$$

where ρ is the mean density of the gas, Re[ ] and Im[ ] represent the real and imaginary components. P_+ and p_- are the positive and negative direction pressure waves, and θ=arg[p_{+1}/p_{-2}] represents the phase lead of p_{+1} relative to p_{-2}. Also, k represents the complex wave number and F is a complex factor. By modified two-sensor method, the values of k and F are[10]:

$$F = 1 - \frac{2J_1(i^{3/2}\sqrt{2}r_0/\delta)}{i^{3/2}(\sqrt{2}r_0/\delta)J_0(i^{3/2}\sqrt{2}r_0/\delta)} \qquad (11)$$

and

$$k = -ik_0\sqrt{\frac{J_0\left(i^{3/2}\sqrt{2}r_0/\delta\right)}{J_2\left(i^{3/2}\sqrt{2}r_0/\delta\right)}}\sqrt{\gamma + (\gamma-1)\frac{J_2\left(i^{3/2}\sqrt{2\sigma}r_0/\delta\right)}{J_0\left(i^{3/2}\sqrt{2\sigma}r_0/\delta\right)}} \qquad (12)$$

where J_n is the nth order complex Bessel function, and σ and γ denote the Prandtl number and the specific heat ratio. The cross-sectional averaged velocity was expressed as:

$$u = \frac{iF}{\omega\rho}\frac{dp}{dx} \qquad (13)$$

where

$$\frac{dp}{dx} = k\frac{p_3 - p_1}{2\sin kdx} \qquad (14)$$

The velocity wave can also be decomposed into positive its and negative direction components.

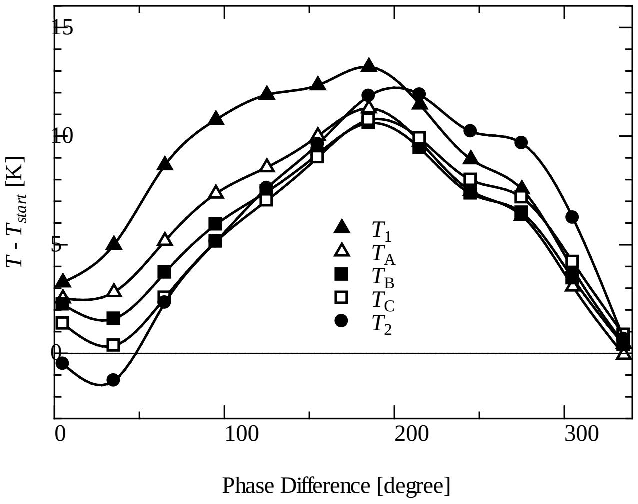

Figure 2 Regenerator temperature difference from start temperature for 250 Hz in relation to travelling wave phase difference

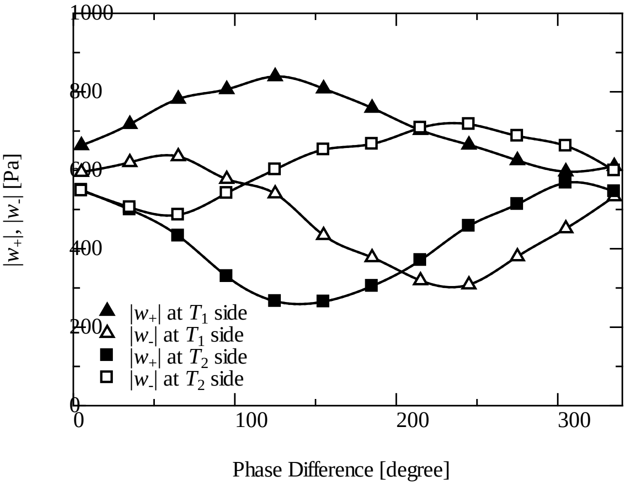

Figure 3 $|w_+|$ and $|w_-|$ at the T_1 and T_2 sides of the regenerator at 250 Hz in relation to travelling wave phase difference

Results and Analysis

The diagram of the relation of temperature change to phase difference at 250 Hz is shown in Figure 2. The recorded final temperature taken 60 minutes after the acoustic driver is initiated. The results shows that changing phase difference influences regenerator temperature distribution. For small phase difference, temperature distribution is linear but as phase difference becomes large, temperature distribution comes close to U-shaped. According to thermoacoustic theory, heat is pumped opposite to the direction of the propagating travelling wave. Based on the measured temperatures within the regenerator, it can be seen that heat can be pumped in both positive and negative directions. Therefore as we have mentioned previously, it is important to decompose the measured pressure wave into its positive (w_+) and negative (w_-) direction travelling wave components.

The diagram of positive (w_+) and negative (w_-) of both ends of the regenerator for changing phase difference at 250Hz is shown in Figure 3. The mechanism to why the magnitudes of w_+ and w_- differ between the two sides of the regenerator involves the attenuation and amplification of each individual travelling wave[8], and travelling waves passing through the regenerator will experience thermoacoustic energy conversion resulting in amplification or attenuation of the wave depending on whether it propagates against or along the temperature gradient within the regenerator. For example, when the phase difference is 35º, since w_+ is larger than w_- at the T_1 side, w_+ is attenuated and w_- is amplified. At that time, we therefore obtain dominant travelling waves and net acoustic power flow in the positive direction for both sides of the regenerator, and heat is pumped to the negative direction.

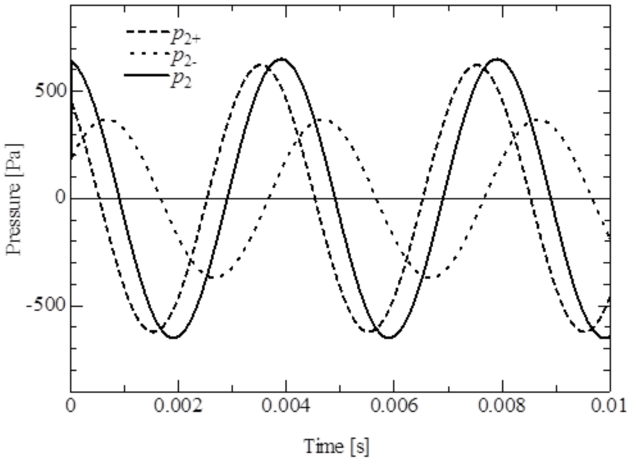

Figure 4 Relation of the measured combined pressure wave and decomposed p_+ and p_- at phase difference 275º for 250 Hz

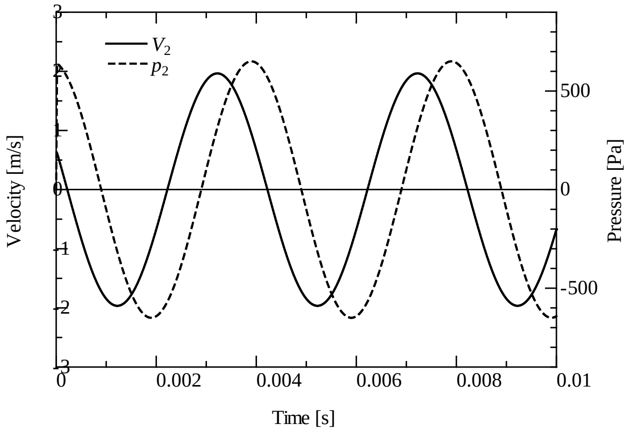

Figure 5 Relation of the measured combined pressure wave and calculated velocity wave at phase difference 275º for 250 Hz

We confirm whether this wave decomposition has decomposed the measured combined pressure wave into the travelling wave. The diagram of the relation of the measured combined pressure wave and decomposed p_+ and p_- at phase difference 275º is shown in Figure 4. This result shows that pressure wave could be decomposed into its positive and negative pressure waves.

The velocity wave has also been calculated by the equation (12). The diagram of the relation of the measured combined pressure wave and calculated velocity wave at phase difference 275º is shown in Figure 5. Obtaining functions of the velocity wave at two points, it is thus possible to decompose the velocity wave into its positive and negative propagating components.

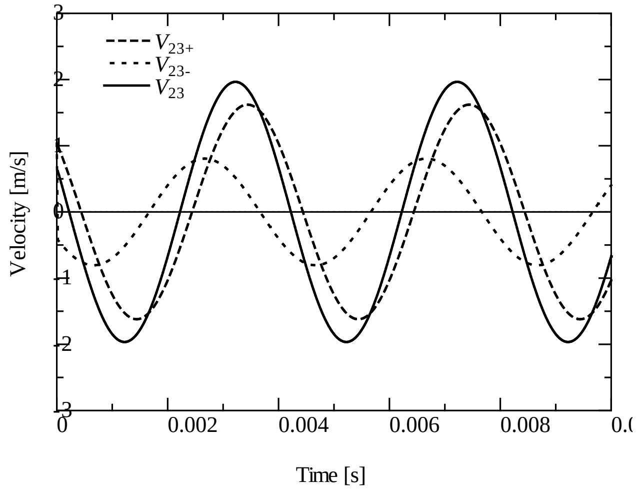

Figure 6 V_{2+} and P_{2+} at the T_1 side of the regenerator at phase difference 275º for 250 Hz

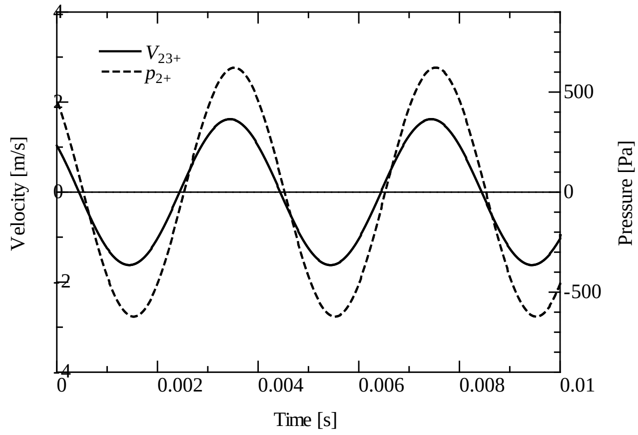

Figure 7 V_{23+} and P_{2+} at the T_1 side of the regenerator at phase difference 275º for 250 Hz

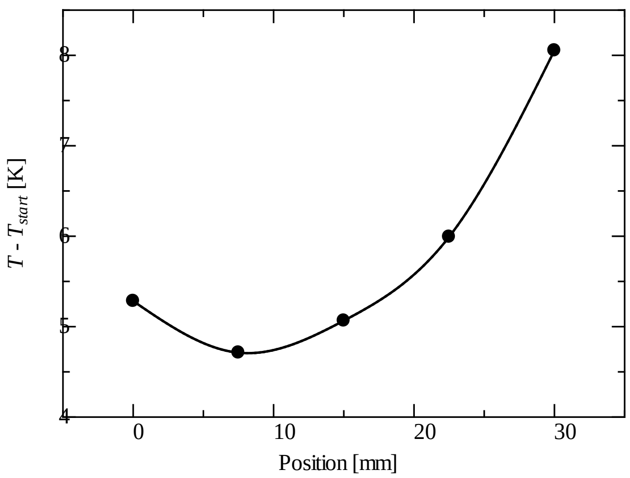

Figure 8 Regenerator temperature difference from start temperature at phase difference 275º for 250 Hz

The diagram of the relation of the calculated velocity wave and decomposed V_+ and V_- at phase difference 275º is shown in Figure 6. This result also shows that velocity wave could be decomposed into its positive and negative velocity waves.

The diagram of the relation of the decomposed pressure and velocity wave is shown in Figure 7. According to theory, since the pressure and velocity of a travelling wave are in-phase, it is apparent that the decomposition has revealed the positive and negative direction travelling waves.

In addition, we have calculated the acoustic power. The calculation is uses the equations (9) to (12). The objective of calculating acoustic power is that we confirm dominant travelling wave as described previously denotes the same tendency of acoustic power. The diagram of the relation of final and initial regenerator temperature distribution at phase difference 275º is shown in Figure 8. 0 mm of a horizontal axis expresses T_1 part. This shows that temperature distribution is U-shaped. When we calculated the acoustic power using the equation, those are 0.61W at the T_1 side and -0.59W at the T_2 side, and net acoustic power flow in the positive direction for T_1 side and the negative direction for T_2 side.

As can be seen in Figure 2, in the entire phase difference range, dominant travelling wave is w+ at the T1 side. , Meanwhile, above 100º the dominant travelling wave is w- at the T2 side. These show that above 100º, the net acoustic power flow in the direction into both ends of the regenerator. When large phase difference relative to small one, the difference of |w+| and |w-| becomes large, and since heat pumped from the central region to both ends of the regenerator is larger, temperature distribution becomes U-shaped.

With heat is pumped opposite to the direction of the acoustic power, the regenerator temperature distribution shows the flow of the acoustic power into both ends of the regenerator. Thus dominant travelling wave as described previously denoted the same tendency of acoustic power.

Conclusions

We have constructed a dual speaker type thermoacoustic and experimented in changing phase difference from 5º to 335º at intervals of 30º at 250Hz to observe the effect of these changes on the acoustic field and heat pumping direction. This phase difference alters the heat transport direction within the regenerator by changing the magnitude and direction of the acoustic field. The direction of the heat pumping was deduced based on the temperature distribution within the regenerator. Results have shown that the temperature distribution changes were indeed caused by acoustic flow changes induced by the change in phase between the two opposite speakers. Based on these results, it may be possible to enhance the cooling effect of a single speaker using a secondary opposite speaker if the phasing and power settings are favorable.

References

[1] A. Tominaga, Fundamental thermoacoustics, *Uchidaroukakuho*, 1998.

[2] G.W. Swift, Thermoacoustic Engines, *Journal of the Acoustical Society of America*, **84**, 1988, 1145-1180.

[3] P.H. Ceperley, A pistonless Stirling engine - The traveling heat wave engine, *Journal of the Acoustical Society of America*, **66**, 1979, 1508-1513.

[4] A. Widyaparaga, T. Koshimizu, E. Noda, N. Sakoda, M. Kohno, Y. Takata, The frequency dependent regenerator cold section and hot section positional reversal in a coaxial type thermoacoustic Stirling heat pump, *Cryogenics*, **51**, 2011, 591-597.

[5] T. Yazaki, T. Biwa, A. Tominaga, A pistonless Stirling cooler, *Applied Physics Letters*, **80**, 2002.

[6] H. Kang, G. Zhou, Q. Li, Thermoacoustic effect of traveling-standing wave, *Cryogenics*, **50**, 2010, 450-458.

[7] T. Biwa, Y. Tashiro, U. Mizutani, M. Kozuka, T. Yazaki, Experimental demonstration of thermoacoustic energy conversion in a resonator, *Physical Review E*, **69**, 2004.

[8] I. Bucher, Estimating the ratio between travelling and standing vibration waves under non-stationary conditions, *Journal of Sound and Vibration*, **270**, 2004, 341-359.

[9] Fusco, A. M, Ward, W. C., and Swift, G.W., Two-sensor power measurements in lossy ducts, *Journal of the Acoustical Society of America*, **91**(4), 1992, 2229-2235.

[10] T. Biwa, Y. Tashiro, H. Nomura, Y. Ueda, T.Yazaki, Acoustic intensity measurement in a narrow duct by a two-sensor method, *Review of scientific instrument*, **78**, 2007, 086110.