Investigation of Geometric Error Management with Respect to Compensatable and Uncompensatable Error on the Three Degree of Freedom Spherical Parallel Mechanism

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Abstract: This paper deals with geometric error management of spherical parallel mechanism. The mechanism is composed of three identical limbs connecting platform to base symmetrically. The error management was done in the dimensional synthesis, mechanical components (joints and links) design, manufacturing, and assembly steps. In the dimensional synthesis was obtained set of kinematic constants of the mechanism to provide large workingspace which is free from singularities. The synthesis was carried out based on the evaluation index (*EV*) generated by velocity transmission and force constraint. The mechanical components design was focused on avoiding the interference between the limbs especially on large inclination angle of platform. Geometric errors physically was maintained in the manufacture and assembly process. The effect of geometric errors and effectiveness of proposed step of design were evaluated experimentally. The achieved workingspace and appearance of uncompensatable error represented by position error of center of platform rotation were used as indicator of effectiveness of using *EV* to design a lower degree of freedom parallel mechanism especially decouple of platform motion. Evaluation of performance proposed mechanism was done based on static condition. Position and orientation error were measured for some postures of the mechanism. Based on the experimental results were clarified that, the mechanism can achieve 45 degree inclination angle of platform relates to 0.85 mm maximum translational error. The mechanism can hold external load equal to weight of all moving parts of the mechanism caused 0.9 mm vertical deflection. Based on the facts, using evaluation index, *EV* is effective to manage the geometric error in the dimensional synthesis of spherical parallel mechanism to yield large workingspace.
Keywords: Kinematics, parallel mechanism, spherical mechanism, workingspace

1. Introduction

Parallel mechanism shows the excellent performance in stiffness and kinematic responses related to its kinematic structure. Platform as output link is connected by several kinematics chain or limbs to base. Such configuration provides high rigidity for small inertia of moving mechanical components. Based on the fact, parallel mechanism has wide range in application such as high precision machine tools and medical equipments. Research in parallel mechanism was developed since introduced the Stewart Platform having six degree of freedom (dof) controlled by six prismatic actuators. In the six dof parallel mechanism the position and orientation workingspace are couple. To control such motion of platform involve s many parameters caused complexity in control system.

In some applications decouple motion of platform is more applicable such as controlling position of work piece and orientation of tool. The pure translation or rotation motion of platform can be produced by lower dof parallel mechanism. Many configurations of lower dof parallel mechanism especially for pure rotational platform motion were developed and proposed. Many configurations of limb were proposed to yield the pure rotational motion of platform [1] – [5]. In these mechanism, motion of center of platform are constrained trans

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lationally by constraint forces produced by three limbs. For special configuration is required additional constraint by connecting center of platform rotation to the fixed body using spherical joint [5],[6] . By giving additional limb caused the structure become more complex and increase number of mechanical components.
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In other point of view, design of pure rotational parallel mechanisms was carried out intensively related to possible application. Overconstrained configuration namely 3-RRR parallel mechanism proposed to provide high speed and precision motion of platform [7]-[9]. To provide such motion the kinematic error must be maintained carefully to avoid motion error of platform. To manufacture the mechanical components of the overconstrained parallel mechanism costly because it is required high precision machining process.

To realize the pure rotational motion of platform without additional constraint on platform and carefully maintain tolerance of mechanical component, in this paper was proposed configuration spherical parallel mechanism composed of platform, base and three limbs arranged symmetrically named as 3-URU (universal-revolute-universal) configuration. The mechanism was design with consideration of workingspace and stiffness. Decision of kinematic constants was based on constraint and actuation singularity. Performance mechanism was evaluated based on orientation achievement and translational error of platform motion. The translational error of platform in pure rotational mechanism cannot be compensated. The source of error caused translational error should be managed carefully. In this paper effect of geometric error were investigated based on simulation and experimental results.

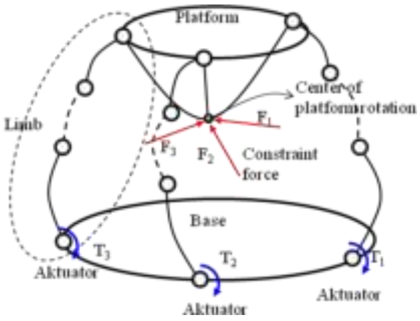
1.1 Basic Configuration of Symmetrical Non overconstrained 5R Spherical Parallel Mechanism

Generally, platform as an output motion to produce spherical motion supports by three limbs. Configuration of limbs can be classified as overconstrained and non overconstrained. In the case of symmetric limbs, overconstrained configuration, the Gruebler-Kutzbah Equation doesn’t satisfy to determine dof of mechanism and the joint axes should be arranged with special requirements. A 3-RRR configuration is one of example of overconstrained spherical parallel mechanisms, where the nine axes of revolute joints must be intersect in a point specified as center of platform rotation. In the case of overconstrained mechanism, geometric tolerance should be managed carefully. On the other hand, for non overconstrained mechanism, the number of intersection joints can be decreased. To define the center of platform rotation at least two joint axes must be intersect for one limb. In other word, there are six joint axes should be used to determine the center of platform rotation.

The mention limbs are constructed by two kinds of basic joint, namely revolute or/and prismatic joints where in the joints are allowed one degree rotational and translational respectively. In some cases, the limbs are used other type of joints giving more degree of freedom to simplify the mechanical structure such as universal, cylindrical, planar and spherical joints.

Typical motion of pure rotational platform is illustrated in Figure 1. The platform can rotate in spatial motion, controlled by three actuators usually located on based and translational motion is constrained by force provided by three limbs. Forces constraint acting on the platform center rotation avoid translational motion of the platform. On the other side, motion of platform is controlled by three actuators located on the base.

(a)



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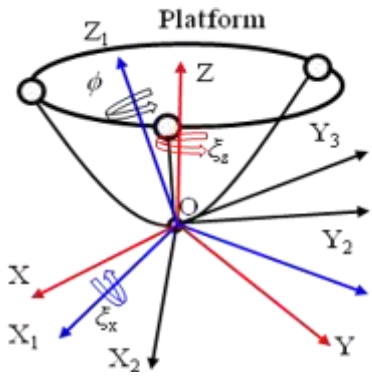

(b)

Figure 1. 3-dof spherical platform motion

1.2 Kinematic Constants of 3-5R Simplified as 3-URU Spherical Parallel Mechanism

Limb structure of spherical parallel mechanism is composed of 5R or its simplification using universal or spherical joint. To produce spherical motion of platform, at least two joint axes have to intersect in one limb. The point determined by intersecting joint axes must coincide for the three limbs to specify center of platform rotation. Based on this consideration will be obtained 28 possible candidates of limb configuration [10]. Based on the reason for simplification of structure, in this paper two revolute joints close to the base and platform simplifies as a universal joints. The schematic diagram of the mechanism is shown in figure 2. Six intersecting joints axes determine location of platform rotation center and three other joint axes are parallel to specify direction of forces constraint.

To simplify, limb composed of 5R was changed to be URU structure shown in Figure 3. Kinematic constants of mechanism are link lengths L_1 and L_2, radius of base and platform, r_B and r_P, angle mounting of joint to base and platform, μ and η. The six kinematic constants were determined based on force constraint on platform and torque produced by actuators transmitted to the platform.

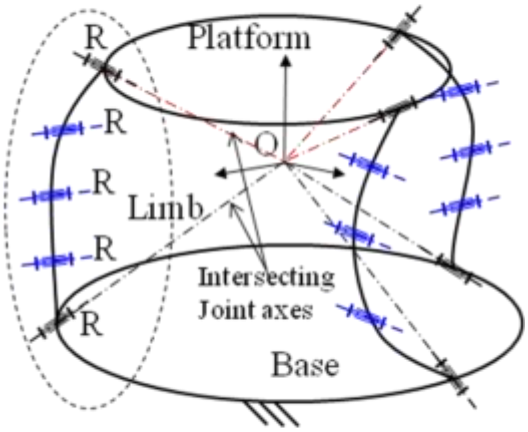

Figure 2. 3-5R spherical parallel mechanism

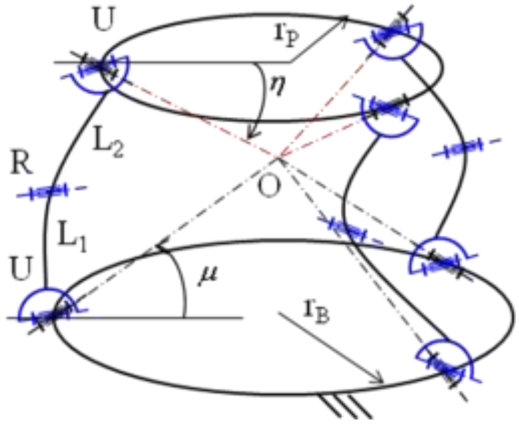

Figure 3. 3-URU spherical parallel mechanism

Determination of six kinematic konstants based on evaluation index (*EV*) obtained from velocity transmision and force constraint. Velocity transmision indicates ability of the three limbs to transmite the velocity from actuator to platform expressed in Equation (1)

$$\dot{\boldsymbol{\theta}} = J_A \dot{\mathbf{x}} \tag{1}$$

$\dot{\theta}$ represents the input joint velocities consisting (3 x 1) matrix, $\dot{x}$ indicates output velocities in (3x1) matrix, and J_A is 3x3 matrix.
On the other hand, forces constraint indicate relationship between internal and external force acting on the platform. The internal forces avoid motion of center of platform rotation. Relationship between internal and eksternal force can be written as

$$\boldsymbol{\tau} = J_C \boldsymbol{f}\,, \tag{2}$$

whrere $\boldsymbol{\tau}, \boldsymbol{f}$ represented vector of internal and external force respectivelly and J_C represents (3x3) matrix.

Evaluation index can be derived using condition determinant matrices J_A and J_c in Equation (1) and (2) expressed as

$$EV = \left|\det J_A \cdot \det J_C\right| \tag{3}$$

Elements of matrix J_{A} and J_{C} are dependent on orientation of platform rotation. Platform orientation expressed using euler angle with three successive rotation angles ξ_Z with respect to Z-Axis, ξ_X with respect to X-Axis and ϕ with respect to Z-Axis (Z-X-Z) system displayed in Figure 1. The rotation matrix can be express as Equation (4)

$$R = \begin{bmatrix} c\xi_z c\phi - s\xi_z c\xi_x s\phi & -c\xi_z s\phi - s\xi_z c\xi_x c\phi & s\xi_z s\xi_z \\ s\xi_z c\phi + c\xi_z c\xi_x s\phi & -c\xi_z s\phi + c\xi_z c\xi_x c\phi & -c\xi_z s\xi_z \\ s\xi_x s\phi & s\xi_x c\phi & c\xi_z \end{bmatrix} \tag{4}$$

where $\mathrm{c}(\cdots) = \cos(\cdots)$, $\mathrm{s}(\cdots) = \sin(\cdots)$.

1.3 Geometrics and Dimensional Error

Geometrics and dimensional errors usually exist in step of manufacturing and assembling of mechanical components. For lower dof parallel mechanism especially for spherical motion of platform such error caused rotational and translational error of platform motion. In case of spherical motion, rotational error can be compensated using control motion of actuators. On the other hand, translational error cannot be compensated after mechanism was assembled. Based on this fact the geometric error should be managed carefully to reduce the magnitude of uncompenstable error.

There are three possibilities appearance of error source in this mechanism before and post assembly. They are (i) error intersecting joint axes, (ii) error in parallel joint axes and (iii) clearance in the joint. Error in intersecting joint axis actually exists because very difficult to maintain at least six joint axis intersect in a common point. For parallel joint axes only maintain for three successive joint axes for each limb. Direction of joint axes indicated direction of constraint force given by each limb. Error in joint clearance give the local mobility of limb with small displacement when actuator lock. Efffect of this error is easy to be observed.

In this paper geometric error is focused on error in intersecting joint axes, because it is difficult to be managed in step of manufacturing and assembling. Two other error sources are easier to be maintained in step of manufacturing and assembling.

1.4 Evaluation of Mechanism Performance

To evaluate performance mechanism, it was designed a prototype of spherical parallel mechanism. Kinematic constants of mechanism was determined using evaluation index, *EV* explained in the section 2. Performance of mechanism is indicated by the nominal of error of platform motion named as translational and rotational errors. The evaluation of errors were carried out as follows:

1.4.1 Evaluation of joint clearance

In this step, the input joint is lock, and then small magnitude of force is applied to the platform. The appearance of joint clearance is recognized if there is local mobility of the limb with small displacement. The evaluation is done on workingspace which is free from singular point.

1.4.2Compliance effect

Effect of compliant occurs when the mechanism has small motion when force acting on platform by locking actuators. If such force released, the platform moves to previous orientation and position. In this paper effect compliance was investigated by checking deformation of the mechanism under weight of moving parts.

1.4.3Effect of Geometric error

Effect of geometric error was evaluated based on error motion of platform. Platform was moved to some orientations, the translational and rotational error were measured without applying force to platform. The translational error was observed in the center of platform rotation represented by translational motion of center of platform rotation. The rotational error was identified by checking relationship between orientation of platform and input joint displacements. If the platform keeps horizontally by giving various ϕ the input displacement will be equal theoretically. If the geometric errors occur the input joint displacements will be different for the three actuators.

2. Results and Discussions

In figure 4 is shown the Cad diagram of 3-URU spherical parallel mechanism and the kinematic constants of the mechanism are shown in Table 1. Form the Figure 4 can be seen that intersecting six of joint axes specify the center of platform rotation. For this model when platform is moved to specified orientation in the workingspace, the center of platform rotation will keep in constant position. This condition is used as an indicator of effect of geometric error on the mechanism. If geometric errors occur it will be found translational displacement of center of platform rotation when platform is oriented.

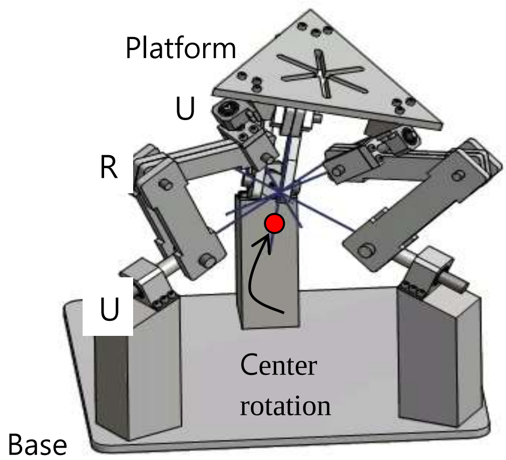

Figure 4. Cad diagram of 3-URU spherical parallel mechanism

Based on the Cad diagram was built a prototype of mechanism. The mechanism uses two kinds of material, steel and aluminium alloys. The moving part which is not connected to base using the light weight material and others use steel. Prototype of the mechanism is depicted in Figure 5. In this figure is shown two orientations of platform. Location of center of platform is shown by the tip of stringger.

Table 1. Dimension and mechanical properties of kinematic constants

r_P (radius of Platform)	110.69 mm
r_B (radius of Base)	172.92 mm
L_3 (upper Link)	140 mm
L_2 (Lower Link)	160 mm
η (angle mounting limb to platform)	55^o
μ (angle mounting limb to base)	22^o

Weight of Platform	1.24 kg
Weight 3 limb (@ = 2.25 kg)	6.75 kg
Total Weight moving components	8 kg

(a) (b)

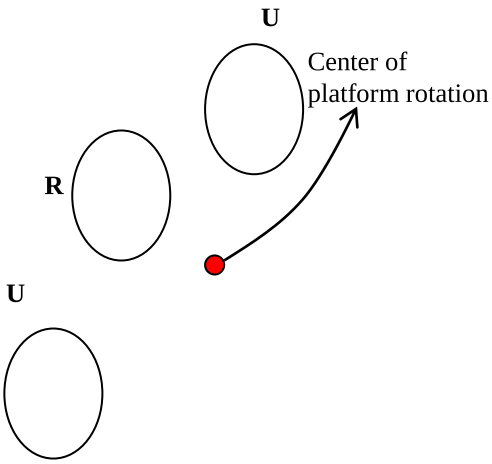

Figure 5 Prototype of 3-URU spherical parallel mechanism

2.1 Identification of actual position of center of platform rotation

Actual location of the center of platform rotation identified experimentally. It is difficult to find exact location of this point, because the geometric error normally exists on the mechanism post assembly. Actual position of the center of platform rotation is obtained by adjusting location of the center of rotation specified from design. For one candidate of the location, the platform is moved to some orientations and at the same time displacement of the center of platform rotation is measured. Then the position of the rotation center is adjusted again and location of the center of platform is measured. Actual position of the center is indicated by minimum displacement of center of platform rotation when platform was oriented in workingspace.
To simplify determination of the location of platform rotation center, It is divided into

ISBN 978 979 8510 61 8

horizontal and vertical position. To specify in horizontal plane, platform rotates horizontally, in this case rotation angle $\xi_x = 0$ with various angle ϕ refers to Figure 1. Location of the center of platform is adjusted until displacement of the candidate rotation center moves small enough. The obtained location is recognized as the center of platform rotation in horizontal plane.
The vertical location, can be obtained by adjusting the location along line perpendicular to platform plane. Platform is oriented freely in workingspace until it is found small enough displacement of candidate of center of platform rotation.

2.2 Stiffness evaluation

The stiffness of mechanism was evaluated in order to check ability of the mechanism to support the force or load. In this paper, evaluation was carried out for static load. The load was given to the platform, then the displacement of center of rotation was measured vertically. Static load applied to platform oriented horizontally equal to weight of moving part, 8 kg. Displacement of platform rotation center was measured for various angle of, ϕ. The displacement of the center of platform rotation in vertical direction is given in Figure 6. Based on the result can be seen that the platform can resist large weight compared to its weight (1 : 1) with the small displacement especially for value of ϕ = 100^o giving good posture of mechanism.

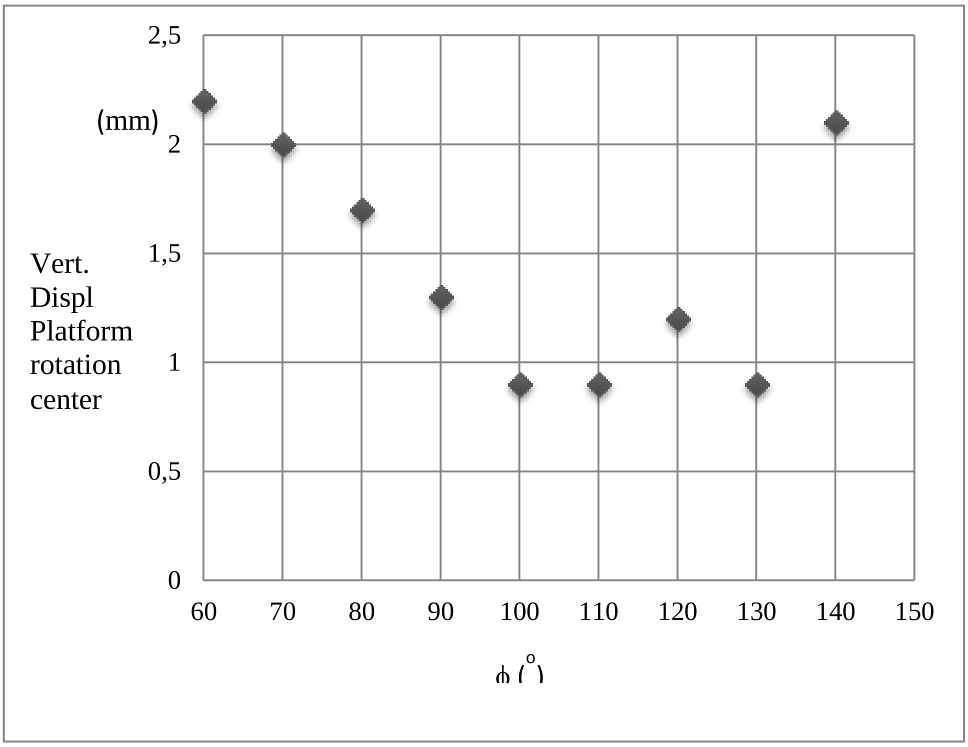

Figure 6 Displacement of the center of platform rotation for various ϕ (W = 8 Kg)

2.3 Uncompensatable Error Evaluation

Two kinds of errors occur on platform motion recognized as translational and rotational errors. Physically, translational error observed on the center of platform rotation considered in steps of design, manufacture and assembly. This error should not occur on the platform motion to keep platform move spherically and it is classified as uncompensatable error because it can`t

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be managed using calibration or control system method.

Different form translational error, rotational error can be compensated using calibration and can be minimized using controlling motion of actuators. These two kinds of errors were investigated for spherical parallel mechanism. The investigation was done experimentally. Each error was evaluated for some orientations of platform.

The first measuring of uncopensatable is carried out for the orientation of platform keep horizontal by taking various angle of rotation along the axis which is perpendicular to the platform plane, ϕ as depicted in Figure 1. The appearance of translational error of platform is shown in Table 2. Based on the result, value of ϕ □can be used to obtain good posture of mechanism that can reduce the error. On the other hand, value of angle ϕ can influence the inclination angle achieved by platform. Large inclination angle is obtained for angle ϕ = 100^o related to 57^o inclination angle of platform. For small value of angle ϕ was achieved small inclination angle of platform because it closes to singular configuration recognized as boundary of prescribed workingspace.

Table 2. Uncompensatable error for horizontal platform orientation with various angle ϕ

No	**Input Angle (^{o})**			**Angle (^{o})**	**Inclination angle of Platform max (^{o})**	**Uncompenstable error (mm)**
	θ1	**θ2**	**θ3**	ϕ	ξ_x	
1	34.5	32.5	34.9	60	18	0.19
2	37.8	37.7	38.9	70	31	0.20
3	41	40.8	41.8	80	46	0.29
4	42.6	42.1	43.6	90	55	0.36
5	43.7	43.4	44.8	100	57	0.35
6	43.7	44.5	46.1	110	53	0.42
7	43.3	44	45.4	120	30	0.55
8	39.3	41	41.6	130	17	0.55

Uncompenstable error is also evaluated for various inclination angle where the angle, ϕ = 100^o is keep constant. The result of measurement is shown in Table 3. Based on the result for large inclination angle error becomes larger. This fact shows that uncompensatable error is large when platform is oriented close to boundary of workingspace.

Table 3. Uncompensatable error for various inclination angle of platform for $\phi = 100^0$

No.	**Input**			**Inclination Angle**	**Uncompenstable error (mm)**
	θ1 (°)	**θ2 (°)**	**θ3 (°)**	ξ_x	
1	10	44	95	45	0.85

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2	15	42	87	35	0.42
3	18	40	70	25	0.65
4	30	39	67	15	0.48
5	41	38	55	5	0.11
6	44	44	44	0	0.35
7	39	45	36	-5	0.51
8	28	50	25	-25	0.39
9	18	55	18	-25	0.51
10	13	60	12	-35	0.45
11	8	80	9	-45	0.82

Effect of geometric error with respect to rotational error can be seen form input which different for three input joint displacement displayed in Table 2. If platform rotate horizontally then the three input joint displacement will be equal. This fact is caused by the limb symmetrically arranged to base and platform.

3. Conclussions

It is obtained a prototype of a spherical parallel mechanism composed of URU configuration. The mechanism can achieve 45^0 inclination angle. Redundant degree of freedom defined as rotation angle along axis perpendicular to the platform plane can be applied to reduced the effect of uncompensatable error. The magnitude of uncompensatable error will increased if platform oriented closed to boundary workingspace. The maximum of magnitude of uncompensatable error for this mechanism is 0.83 mm. For static load the mechanism can resist the external load around one times of weight of moving parts with 0.9 mm displacement. Based on the appearance output motion error and static analysis the evaluation index, *EV* is effective to determine kinematic constants of mechanism.

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Acknowledgement

This paper was written based on partial result of the research funded by Hibah Penelitian Fundamental Universitas Andalas Contract No. 27.UN.16/PL-FD/2013 through DIPA Universitas Andalas No. 023.04.2.415061/2013, 05 December 2013.